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Kohei Hasui

Yoshiyuki Nakazono

Yuki Teranishi

UTokyo Price Project
702 Faculty of Economics, The University of Tokyo,
7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan
Tel: +81-3-5841-5595
E-mail: watlab@e.u-tokyo.ac.jp
http://www.price.e.u-tokyo.ac.jp/english/

Role of Expectation in a Liquidity Trap*

Kohei Hasui† Yoshiyuki Nakazono‡ Yuki Teranishi§

Abstract

This paper investigates how expectation formation affects monetary policy effectiveness in a liquidity trap. We examine two expectation formations: (i) different degrees in anchoring expectation and (ii) different degrees in forward-lookingness to form expectation. We reveal several points as follows. First, under optimal commitment policy, expectation formation for an inflation rate does not markedly change the effects of monetary policy. Second, contrary to optimal commitment policy, the effects of monetary policy significantly change according to different inflation expectation formations under the Taylor rule. The reductions to an inflation rate and the output gap are mitigated if the expectation is well anchored. This rule, however, can not avoid large drops when the degree of forward-lookingness to form expectation decreases. Third, a simple rule with price-level targeting shows some similar outcomes according to different expectation formations as the Taylor rule does. However, in a simple rule with price-level targeting, an inflation rate and the output gap drop less severe due to a history dependent easing and are less sensitive to expectation formations than in the Taylor rule. Even for the Japanese economy, the effects of monetary policy on economic dynamics significantly change according to expectation formations for rules except optimal commitment policy. Furthermore, when the same expectation formations for the output gap are assumed, we observe similar outcomes.

JEL Classification: E31; E52; E58; E61

Keywords: Expectation; Liquidity Trap; Monetary Policy

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†Matsuyama University. E-mail: khasui@g.matsuyama-u.ac.jp

‡Yokohama City University. E-mail: nakazono@yokohama-cu.ac.jp

§Keio University. E-mail: yukitera@fbc.keio.ac.jp

1 Introduction

Expectation is one of the most important factors in the conduct of monetary policy. In particular, managing expectation of an agent is a nontrivial tool in a liquidity trap since the central bank faces limitation in reducing the policy interest rate. A lot of papers, such as Eggertsson and Woodford (2003b), Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006, 2007), and Nakov (2008), analyze optimal monetary policy in a liquidity trap and conclude that optimal commitment policy is very effective. The commitment policy can reduce the real interest rate and stimulate the economy by controlling the inflation expectation. Their conclusions, however, are solely dependent on two important assumptions, i.e., rational expectation and optimal commitment monetary policy.

In this paper, we relax an assumption of purely forward-looking rational expectation and show a role of expectation formation in a liquidity trap under a standard New Keynesian model. We assume two states in expectation formation. First, we change the degree of how much expectation is anchored. In particular, some papers argue that inflation expectation is well anchored under an inflation targeting policy. Beechey, Johannsen, and Levin (2011) show that long-run inflation expectations in the euro area are well anchored. In the United States, however, the expected inflation rate is not firmly anchored. Such a difference comes from the ECB's communication strategy in which a goal of price stability is specified by a number of a target inflation rate. Using survey data of the inflation expectation for 36 developed and developing countries, Davis (2014) argues that inflation expectation tends to be anchored for periods after introducing inflation targeting policy. We describe this situation simply by fixing a fraction of expected inflation at a target level. Second, we change a degree of how much expectation formation is forward-looking and describe it by assuming that expectation depends on a weighted average between rational expectation and a current inflation rate. Numerical simulations reveal how these different expectation formations change the effects of monetary policy in a liquidity trap.

Moreover, we also relax an assumption of optimal commitment policy. In addition to optimal commitment policy, we introduce two realistic monetary policy rules, i.e., the Taylor rule and a simple rule with price-level targeting. Several papers, such as Smets and Wouters (2003) and Smets and Wouters (2007), assume Taylor rules to fit a theoretical model to data. Regarding a simple rule with price-level targeting, Eggertsson and Woodford (2003b) and Eggertsson and Woodford (2003a) reveal that this rule is very effective in a liquidity trap due to history dependent monetary easing. They argue that price level targeting policy is a simple and realistic rule to replicate a feature of optimal monetary policy.

We obtain significantly different outcomes according to monetary policy rules in numerical simulations. Under optimal commitment policy, the effect of monetary policy does not change markedly with different expectation formations for an inflation rate. A reason for this is that optimal monetary policy includes a feature of history dependent easing and so can manage expectation significantly even though the room for managing expectation is limited. Thus, the role of expectation formation for an inflation rate is not so important for optimal monetary policy. On the other hand, under the Taylor rule, reductions of an inflation rate and the output gap for firstly some periods become sufficiently smaller as the degree of anchoring the inflation expectation becomes stronger. Also, drops in an inflation rate and the output gap become sufficiently smaller as the degree of forward-lookingness in expectation formation become stronger. The Taylor rule does not hold a history dependency and does not work on expectation in a forward-looking model. Thus, anchored expectation and forward-looking expectation compensate for a drawback of the Taylor rule. In a forward-looking economy, there is a force to realign the economy to a steady state where a negative shock disappears. Thus, when the degree of forward-lookingness decreases, such a force weakens and an economic slowdown is severe. The role of expectation formation is non trivial under the Taylor rule. A simple rule with price-level targeting can mitigate large drops in an inflation rate and the output gap. As explained in previous papers analyzing monetary policy in a liquidity trap, this is because targeting a price-level gives a policy maker control of expectation formation by promising a future monetary easing as optimal commitment policy does. This is still effective even though room for managing expectation is limited.

Moreover, unlike the Taylor rule, an inflation rate and the output gap are less sensitive to expectation formations. Thus, the role of expectation formation is no so serious problem for a simple rule with price-level targeting.

In particular, we estimated the degree of how much expectation is anchored to a targeting level and the degree of how much expectation formation is forward-looking for the Japanese economy. In Japan, expectation is partially anchored after the Bank of Japan introduces a price stability target of 2 percent in January 2013. Moreover, expectation formation is not perfectly forward-looking and depends on the present inflation rate, which implies adaptive expectation. However, even for the Japanese case, expectation formation is not a topic in monetary policy regardless of expectation being anchored or not if the Bank of Japan can implement optimal commitment policy. Optimal monetary policy with strong history dependent easing can control expectation formation in the Japanese economy. Under the Taylor rule, compared to a perfectly anchored case, we observe drops in an inflation rate and the output gap for other expectation formations for an inflation rate. However, these drops are largely mitigated when an inflation expectation is partially anchored. Thus, even under a weak anchoring in an inflation expectation, the Taylor rule can mitigate a serious deflation. These drops are larger in a case where an inflation expectation is based on a current inflation rate in compared to a case where an inflation expectation is partially anchored. Moreover, by committing to a simple history dependent rule like a price-level targeting rule, the Bank of Japan can further mitigate an effect of a weak anchoring for expectation and a lack of forward-lookingness in expectation formation.

Furthermore, we assume a situation where a government makes expectation for the output gap by using tools such as government expenditure and a tax cut in the future. A fiscal stimulus is necessary for some cases to escape from a liquidity trap and expectation for fiscal policy plays important roles in a liquidity trap as shown in Reifschneider and Williams (2000), Eggertsson (2008), and Werning (2011). We introduce the same expectation formations for the output gap as for an inflation rate. As in cases of expectation formations for inflation rates, the effect of monetary policy does not change markedly for different expectation formations for the output gap under optimal commitment policy. For the Taylor rule, reductions of an inflation rate and the output gap become sufficiently smaller as the degree of anchoring the future output gap becomes stronger. Also, drops in an inflation rate and the output gap become sufficiently smaller as the degree of forward-lookingness in expectation formation become stronger. Under a simple rule with price-level targeting, drops in an inflation rate and the output gap are mitigated in any expectation formation in comparison to those under the Taylor rule. These results imply that a government rather than a central bank can contribute to escape from a liquidity trap by managing expectation for the future output gap.

Our paper is related to two strands of previous literature. First, our paper is related to optimal monetary policy in a liquidity trap. Eggertsson and Woodford (2003b) and Jung, Teranishi, and Watanabe (2005) analyze the optimal commitment policy in a liquidity trap and show that a central bank needs to continues a zero interest rate policy even after the natural rate turns positive. Adam and Billi (2006, 2007) and Nakov (2008) analyze the optimal commitment policy and discretionary policy in a liquidity trap under a stochastic shock.[1] Our paper relaxes the assumption of forward-looking rational expectation in these papers and analyzes the role of expectation formation in a liquidity trap.

Second, our paper is related to formation of expected inflation with empirical assessment and a forward guidance puzzle. Recently, a sluggish response of an expected inflation rate has been described in the New Keynesian model. Coibion and Gorodnichenko (2012, 2015) show the state of a sluggish response in expected inflation rate using the U.S. survey data. Pfajfar and Žakelj (2016) introduce the expectation formation based on a laboratory experiment into the New Keynesian model and analyze the

[1] A lot of papers analyze monetary policy in a liquidity trap. Werning (2011) analyzes fiscal policy and monetary policy in a liquidity trap in a continuous New Keynesian model. Billi (2011) analyzes the optimal long-run inflation rate of U.S. in a stochastic framework with robust control technique. Fujiwara, Nakajima, Sudo, and Teranishi (2013) extend the model to the open economy and show an optimal zero interest rate policy in a global liquidity trap.

design of monetary policy when the expectation is not perfectly rational.[2] Wiederholt (2014) and Andrade, Gaballo, Mengus, and Mojon (2015) built up a model including the sluggish response of an expected inflation rate by extending the New Keynesian model with heterogeneous belief. Some papers focus on expectation formation to solve the forward guidance puzzle that is pointed out by Del Negro, Giannoni, and Patterson (2012). They show that the a forward guidance is unrealistically powerful in the New Keynesian model. For example, Andrade, Gaballo, Mengus, and Mojon (2015) show that pessimistic expectation weakens the effects of forward guidance. Our paper is related to these papers in focusing on effects of expectation formation on monetary policy and economic dynamics.

The remainder of the paper is structured as follows. In Section 2, we define expectations formation and show the empirical evidence. We set up the model in Section 3. Section 4 shows numerical results for expectations formation of an inflation rate. In Section 5, we calibrate a model for Japanese economy and show effects of expectations formation on monetary policy in Japan. Section 6 shows numerical results for expectations formation of the output gap. Section 7 concludes the paper.

2 Expectations Formation and Empirical Evidences

Before examining the theoretical model and numerical simulations, this section discusses how an assumption of purely forward-looking rational expectation is relaxed and provides some evidences supporting the variants of expectation formation. Figure 1 and Figure 2 show longer-term inflation forecasts (5 to 10-year ahead forecasts of Consensus Forecasts from Consensus Economics) and short-term inflation forecasts (1-year ahead forecasts of Consensus Forecasts from Consensus Economics), respectively. These figures suggest that an inflation expectation is partially anchored by 2 percent target level set by the

[2]The approach of introducing expectation from the laboratory is also shown in Marimon and Sunder (1994) and Bernasconi and Kirchkamp (2000) for analyzing the effects of monetary policy. Recently, Adam (2007) shows persistent responses of output and the inflation rate by introducing the expectation from the experiment in a laboratory to a simple cash-in-advanced model.

Bank of Japan in January 2013 or partially depends on a current inflation rate.

We assume two cases for expectation formations for the inflation rate as follows.

$$\pi^e_{t+1} := \begin{cases} \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1-\gamma_\pi)\bar{\pi}, & \text{(a)} \\ \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1-\gamma_\pi)\pi_t, & \text{(b)} \end{cases} \tag{1}$$

where γ_π is a parameter satisfying $0 \leq \gamma_\pi \leq 1$.

In a case of (a), we change a degree of how much expectation is anchored to a targeting level set by a central bank in an inflation targeting policy. Expected inflation rate is given by a weighted average between a rational inflation expectation and an anchored inflation at a constant number. When γ_π is one, expectation formation follows a rational expectation as in a standard New Keynesian model. On the other hand, when γ_π is zero, expectation for a future inflation rate is strongly anchored at constant $\bar{\pi}$.

In a case of (b), we change a degree of how much expectation formation is forward-looking and describe it by assuming that expectation depends on a weighted average between rational expectation and a current inflation rate. In this case, economic agents reflect current inflation to form an inflation expectation. As γ_π decreases, a degree of forward-lookingness in expectation formation decreases. Note that the inflation expectation π^e_{t+1} is reduced to perfectly forward-looking, i.e. $\pi^e_{t+1} = \mathrm{E}_t\pi_{t+1}$ when $\gamma_\pi = 1$.

We estimate the degrees and examine whether these assumptions are empirically supported. In order to estimate γ_π, we transform equation (a) into the following equation by using the definition of the forecast error,[3]

$$\pi_{t,t+k} - \pi^e_{t,t+k} = \beta(\pi^e_{t,t+k} - \bar{\pi}) + \varepsilon_{t,t+k}, \tag{2}$$

where

$$\beta = \frac{1-\gamma_\pi}{\gamma_\pi},$$

and

$$\varepsilon_{t,t+k} = \pi_{t,t+k} - \mathrm{E}_t\pi_{t,t+k}.$$

$\pi^e_{t,t+k}$ is defined as an inflation expectation over k-periods ahead and is formed at time t. $\varepsilon_{t,t+k}$ denotes the forecast error and should not be predictable from information in time

[3]This estimation strategy follows Ichiue and Yuyama (2009).

t under rational expectations. As a result, we can test whether $\beta = 0$. When $\gamma_\pi = 1$, expectation formation follows a rational expectation as in a standard New Keynesian model. When $\gamma_\pi < 1$, agents put some weight on $\bar{\pi}$. Following the introduction of "Price Stability Target" of 2 percent by the Bank of Japan in January 2013, we set $\bar{\pi} = 2\%$ after January 2013 and $\bar{\pi} = 1\%$ before that in equation (a). By rewriting equation (2), we estimate the following equation:

$$\pi_{t,t+k} - \pi^e_{t,t+k} = \beta^A(\pi^e_{t,t+k} - 1\%) \times D + \beta^B(\pi^e_{t,t+k} - 2\%) \times (1 - D) + \varepsilon_{t,t+k}, \tag{3}$$

where a dummy variable D takes one before 2013, otherwise zero. When estimating equation (3), we set k to be four and the inflation expectation, $\pi^e_{t,t+4}$,[4] is quarterly forecast on inflation rates over four-quarter ahead about Japan at time t. Thus, one period in the equation corresponds to one quarter. The data on inflation expectations is obtained from Consensus Forecast, collected by Consensus Economics.[5] We use the year-on-year rate of change in the CPI (excluding perishables) at time $t + 4$ as $\pi_{t,t+4}$. The data covers from 1994:Q1 to 2016:Q2.

Table 1 shows the estimation results in equation (3), namely β^i and $\gamma^i_\pi = \frac{1}{\beta^i+1}$ ($i = A, B$). While γ^A_π is almost one before the introduction of an inflation target, γ^B_π becomes statistically non-zero as approximately 0.64 in a 10 percent interval after that. It is suggested that inflation expectations are weakly and partially anchored after the new inflation target at 2% is introduced in January 2013.[6] This evidence is consistent with the literature which documents unstable inflation expectations in recent years.[7]

Next, in order to estimate γ_π in case (b), we arrange equation (b) into the following

[4]When k is set to be four, $\pi^e_{t,t+4}$ corresponds to inflation forecasts over one-year ahead.

[5]Consensus Forecast (CF) is one of the longest surveys regarding inflation expectations in Japan. CF is a monthly survey, published by Consensus Economics, on developed and developing countries for professional forecasters such as economists. CF publishes quarterly forecasts in the first month of each quarter. As for inflation outlook, forecasters submit year-on-year changes for the CPI (all items).

[6]Figure 1 provides another evidence that even longer-term inflation forecasts are not anchored to 2% after 2013.

[7]Łyziak and Paloviit (2017), Nautz and Strohsal (2013), and Strohsal, Melnick, and Nautz (2016) report that inflation expectations are de-anchored after the onset of a global financial crisis.

equation,

$$\pi_{t,t+k} - \pi^e_{t,t+k} = \beta(\pi^e_{t,t+k} - \pi_{t-k,t}) + \varepsilon_{t,t+k}, \tag{4}$$

where

$$\beta = \frac{1-\gamma_\pi}{\gamma_\pi},$$

and

$$\varepsilon_{t,t+k} = \pi_{t,t+k} - \mathrm{E}_t\pi_{t+k}.$$

$\varepsilon_{t,t+k}$ also denotes the forecast error and should be white noise from information set in time t under rational expectations. As a result, we can test a null hypothesis of $\beta = 0$. In estimating equation (4), k is set to be four.

Table 2 shows the estimation results in equation (4), namely β and $\gamma_\pi = \frac{1}{\beta+1}$. The estimate of γ_π is approximately 0.8; expectation formation basically follows rational expectation, but forecasters put small weight on realized inflation rates at time t ($\pi_{t-4,t}$). This indicates that when expectations are formed, an adaptive response to *current* inflation rates impedes the formation of rational expectations.[8]

3 Model Setup

3.1 Model

The model is a New Keynesian model proposed by Woodford (2003). The macroeconomic structure is given by following three equations.

$$x_t = x^e_{t+1} - \sigma(i_t - \pi^e_{t+1} - r^n_t), \tag{5}$$

$$\pi_t = \beta\pi^e_{t+1} + \kappa x_t, \tag{6}$$

$$r^n_t = \rho_r r^n_{t-1} + \epsilon_t, \tag{7}$$

where x_t, π_t, i_t, and r^n_t denote output gap, inflation rate, nominal interest rate, and natural interest rate, respectively. For arbitrary variable z, z^e denotes the expectation

[8]Figure 2 suggests that realized inflation rates and inflation forecasts are closely related to each other. This also implies that inflation forecasts are affected by the most recent inflation rates.

of z. ϵ_t denotes an i.i.d. disturbance with standard deviation σ_ϵ. σ, κ, and ρ_r are parameters satisfying $\sigma > 0$, $\kappa > 0$, and $0 \leq \rho_r < 1$. Equation (5) is a forward looking IS curve, which is derived by households' intertemporal decision for consumption. Equation (5) shows that the current output gap depends on an expected output gap and deviation of the real interest rate from the natural interest rate. Equation (6) is the New Keynesian Phillips curve (henceforth, NKPC), which is derived by firms' optimal price setting with price stickiness. Equation (6) shows that the current inflation rate depends on the current output gap and expected inflation rate.[9]

The slope of equation (6), κ consists of deep parameters as follows.

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\sigma^{-1}+\omega}{1+\omega\theta},$$

where α, ω and θ denote the rate of fixing price, elasticity of marginal cost, and elasticity of demand for goods.

In this paper, we assume three cases for monetary policy rules, i.e., optimal commitment policy, the Taylor rule, and a simple rule with price-level targeting.

First, we describe a case of optimal commitment policy. The central bank minimizes the intertemporal loss function.

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda_x x_t^2), \tag{8}$$

where $\lambda_x \equiv \kappa/\theta > 0$. The central bank faces a nonnegativity constraint on the nominal interest rate.

$$i_t \geq 0. \tag{9}$$

[9]Note that this Phillips curve becomes a discounted Phillips curve in a sense that a parameter for an expected inflation rate is discounted by γ_π when expectation formation is partially anchored to a targeting level, i.e., a case of (a). When expectation formation depends on a weighted average between rational expectation and a current inflation rate, i.e., a case of (b), this Phillips curve again becomes a discounted Phillips curve. In this case, a parameter for the output gap becomes greater than κ. See Gabaix (2016) for a different justification for a discounted Phillips curve.

The central bank minimizes an intertemporal loss function (8) subject to equations (5), (6), and (9). First order conditions under optimal commitment policy are as follows.[10]

$$\pi_t - \beta^{-1}\sigma\phi_{1t-1} + \phi_{2t} - \phi_{2t-1} = 0, \tag{10}$$

$$\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0, \tag{11}$$

$$i_t\phi_{1t} = 0,\ \ \phi_{1t} \geq 0,\ \ i_t \geq 0, \tag{12}$$

where ϕ_{1t} and ϕ_{2t} denote Lagrange multipliers associated with equations (5) and (6). Equations (10) and (11) show first order conditions with respect to the inflation rate and output gap, respectively. Equation (12) shows the first order conditions with respect to the nominal interest rate considering the nonnegativity constraint. If the nominal interest rate is zero, the Lagrange multiplier ϕ_{1t} becomes positive and vice versa.

Second, we define the Taylor rule. We set the following interest rate rule with the nonnegativity constraint on the nominal interest rate.

$$i_t = \max[0, \psi_\pi \pi_t + \psi_x x_t], \tag{13}$$

where ψ_π and ψ_x are parameters satisfying $\psi_\pi > 0$ and $\psi_x > 0$.

Third, we introduce a simple rule with price-level targeting as follows.

$$i_t = \max[0, \psi_p(\ln P_t - \ln P^*) + \psi_x x_t], \tag{14}$$

where $\ln P^*$ is steady state value of $\ln P_t$ and $\psi_p > 0$.

3.2 Baseline Calibration

We set baseline quarterly parameters as in Table 3. We set $\sigma = 6.25$ following Woodford (2003). We set $\alpha = 0.875$, $\beta = 0.995$, $\omega = 2.149$, $\theta = 6.0$, $\sigma_\epsilon = 0.102$, and $\rho_r = 0.892$ following Sugo and Ueda (2008) that estimate parameters for the Japanese economy.[11]

[10] The first order conditions are derived by supposing $z^e_{t+1} = \mathrm{E}_t z_{t+1}$. Even when we explicitly reflect expectation formations in first order conditions, results in this paper do not change quantitatively as shown in Appendix A.

[11] Sugo and Ueda (2008) estimate preference shock for the natural rate shock. We use the estimated value of preference shock as that of the natural rate shock. Regarding σ, a smaller value can not secure

For a baseline calibration, we set the parameter of weights on an inflation expectation as $\gamma_\pi = 0.5$ and we set anchored levels of inflation rate as $\bar{\pi} = 0$ and the output gap as $\bar{x} = 0$.

4 Baseline Simulations

In this section, we reveal the role of expectation in a liquidity trap by numerical simulations following expectation formation for inflation rate as shown in equation (1). We show cases under different parameters for expectation formation and different monetary policies.

4.1 Optimal Commitment Policy

We assume that a one-time shock of natural interest rate occurs at period 0.[12] We give a negative 0.75 percent (annually 3 percent) quarterly shock to a natural interest rate. Figure 3 shows impulse responses under optimal commitment policy. Solid lines denote a case when an inflation expectation is purely forward-looking i.e., $\pi^e_{t+1} = \mathrm{E}_t\pi_{t+1}$. Dashed lines denote a case when the inflation expectation is perfectly anchored i.e., $\pi^e_{t+1} = \bar{\pi} = 0$. Chained lines denote a case where the inflation expectation is partly anchored, i.e., $\gamma_\pi \mathrm{E}_t\pi_{t+1} + (1-\gamma_\pi)\bar{\pi}$. Dotted lines denote a case in which a degree of forward-lookingness decreases i.e., $\gamma_\pi \mathrm{E}_t\pi_{t+1} + (1-\gamma_\pi)\pi_t$, where $\gamma_\pi = 0.5$.

Figure 3 shows some observations.[13] The response of the inflation rate, the output gap, nominal interest rate, and real interest rate do not change so much according to inflation expectation formation (Figure 3a, b, e, f). Note that responses of the nominal interest rate and real interest rate are mostly identical for all cases. These observations

a convergence in simulation in a liquidity trap. Even when we set $\sigma = 2$ that is a closer value to that in Sugo and Ueda (2008), a conclusion in this paper does not change as shown in Appendix A.

[12] The details of the numerical algorithms are given in Appendix B.

[13] Evaluations from welfare losses are shown in Appendix A. Moreover, a frequency of hitting a zero lower bound on a nominal interest rate differs according to monetary policy rules as shown in Appendix A.

show that, under optimal commitment policy, the formation of an inflation expectation is a trivial problem. A power of commitment is a key point for this result. Under optimal commitment policy, power of controlling the expectation is strong due to a history dependent monetary easing. Thus, a response of an economy does not change drastically even when the room for managing expectation is limited by anchored inflation expectation and by less forward-looking inflation expectation.[14]

4.2 The Taylor Rule

Next, we look at impulse responses under the Taylor rule. We set $\psi_\pi = 1.5$ and $\psi_x = 0.5$. Figure 4 shows results. Responses drastically change in comparison to those under optimal commitment policy in two points.

First, in cases of perfectly and partly anchored inflation expectation, monetary policy achieves smaller drops in the inflation rate and the output gap (dashed lines and chained lines in Figure 4a, b). A reason for this result is given by a low real interest rate. The real interest rate stays at a lower level as shown in Panel (f) in Figure 4 since an inflation expectation is anchored. Consequently, monetary policy can avoid large drops in an inflation rate and the output gap. Thus, under the Taylor rule, anchoring an inflation expectation plays an important role to stabilize the economy. In other words, the effects of monetary policy significantly change according to different inflation expectation formations under the Taylor rule. This is because the Taylor rule does not hold a history dependency and can not work on expectation in a forward-looking model. An anchored expectation can compensate for a drawback of the Taylor rule.

How much degree of anchoring is needed for stabilizing the economy can still be an important point. To answer this question, we show impulse responses by changing γ_π in $\pi^e_{t+1} = \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1-\gamma_\pi)\bar{\pi}$. Figure 5 plots the responses under the Taylor rule for $\gamma_\pi =$ 0, 0.4, 0.8, and 1. As γ_π decreases, an inflation expectation is more strongly anchored.

[14] We analyze a case when expectation formation follows $\pi^e_{t+1} = \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1-\gamma_\pi)\pi^e_t$. We obtain more persistent responses of the inflation rate, but responses of the output gap, nominal interest rate, and real interest rate do not sufficiently change.

A result shows that the output gap becomes larger and the real interest rate smaller as γ_π becomes larger. An important point is that drops in an inflation rate and the output gap are sufficiently mitigated even for the small weight of an anchored inflation rate such as $\gamma_\pi = 0.8$. This means that the benefit of anchoring inflation expectation exists even though the inflation expectation is not anchored strongly. Thus, partly anchoring an inflation expectation is effective to stabilize an economy in a liquidity trap.

Second, an impulse response changes when a degree of forward-lookingness in expectation formation changes. The output gap and inflation rate decrease more as the degree of the forward-lookingness becomes smaller. Panels (a) and (b) in Figure 4 indicate this observation: Dotted lines sufficiently decrease in all responses. When a partial inflation expectation reflects a current inflation rate, monetary policy faces difficulty in controlling economic dynamics since the Taylor rule can not work on expectation in a forward-looking model. Forward-looking expectation can compensate for a drawback of the Taylor rule. In a forward-looking economy, there is a force to align an economy to a steady state where a negative shock disappears. When the degree of forward-lookingness decreases, such a force weakens.

Furthermore, we investigate the effectiveness of interest rate smoothing against a low degree of forward-lookingness in expectation by introducing a lagged interest rate into the Taylor rule. We modify the interest rate rule (13) to the following rule with persistence ρ_i.

$$i_t = \max[0, \rho_i i_{t-1} + (1 - \rho_i)(\psi_\pi \pi_t + \psi_x x_t)], \tag{15}$$

where we set $\rho_i = 0.5$. Figure 6 shows the responses under a low degree of forward-lookingness in expectation formation. Solid lines denote a case without a lag of interest rates and dashed lines denote a case with a lag of interest rates. A result shows that drops in an inflation rate and the output gap are smaller under the Taylor rule with lagged interest rates than under the Taylor rule without lagged interest rates (Figure 6a, b). This means that the Taylor rule with an interest rate inertia can mitigate reductions of an inflation rate and the output gap in a liquidity trap. With interest rate smoothing, agents expect a low interest rate to continue longer into the future. It decreases a real interest rate, and, consequently, stimulates an inflation rate and the output gap. This mechanism is the same as in the case of optimal commitment policy.

4.3 Simple Rule with Price-level Targeting

Some previous papers, such as Eggertsson and Woodford (2003b), Nakov (2008), and Fujiwara, Nakajima, Sudo, and Teranishi (2013), show that a simple rule with price-level targeting is effective in a liquidity trap. We investigate the effectiveness of a simple rule with price-level targeting under different inflation expectation formation. In simulations, we set $\psi_p = 1.5$.

Figure 7 shows impulse responses. As observed in previous results for the Taylor rule, the response of an economy changes according to expectation formation for an inflation rate (Figure 7b). However, compared to a case of the Taylor rule, all responses do not change markedly (Figure 7a, e, f). Although the output gap significantly decreases, the output gap is still higher under a simple rule with price-level targeting than under the Taylor rule. Though inflation rates show some differences, the output gaps are not so different according to expectation formation. Under a simple rule with price-level targeting, targeting a price-level gives power to control the expectation formation due to history dependent easing. Thus, the expectation formation is not so serious problem for a simple rule with price-level targeting.

5 Simulations for the Japanese Economy

In this section, we use estimated parameters for expectation formations for the Japanese economy in section 2.

We show impulse responses under the estimated value of γ_π in Tables 1 and 2. We assume three cases for monetary policy as optimal commitment policy, Taylor rule, and a simple rule with price-level targeting. Figures 8, 9, and 10 show impulse responses under commitment policy, the Taylor rule, and a simple rule with price-level targeting, respectively. Dashed lines in figures denote the case when the inflation expectation is perfectly anchored. Chained lines denote the case when $\gamma_\pi = 0.643$ in $\pi^e_{t+1} = \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1 - \gamma_\pi)\bar{\pi}$. Dotted lines denote the case when $\gamma_\pi = 0.803$ in $\pi^e_{t+1} = \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1 - \gamma_\pi)\pi_t$.

Figure 8 shows that responses of the inflation rate, the output gap, and interest rates do not change so much under the commitment policy. This implies that expectation formation is not a topic in monetary policy if the Bank of Japan can implement optimal commitment policy. Optimal monetary policy with strong history dependent easing can control expectation formation in the Japanese economy. Figure 9 shows a case of the Taylor rule. Compared to a perfectly anchored case, we observe drops in the inflation rate and the output gap for other expectation formations for the inflation rate. However, these drops are largely mitigated when an inflation expectation is partially anchored by approximately 36 percent. Thus, even under a weak anchoring in an inflation expectation, the Taylor rule can stop a serious deflation though optimal monetary policy and a simple rule with price-level targeting show much better outcomes than the Taylor rule does. Moreover, these drops are larger in a case where an inflation expectation is based on a current inflation rate by approximately 20 percent compared to a case where an inflation expectation is partially anchored. Figure 10 shows a case of a simple rule with price-level targeting. Under a simple rule with price-level targeting, differences in impulse responses are smaller than under the Taylor rule. Therefore, by committing to a simple history dependent rule like a price-level targeting rule, the Bank of Japan can further mitigate an effect of a weak anchoring in expectation and a lack of forward-lookingness in expectation formation.

6 Discussion: Role of Output Gap Expectation in a Liquidity Trap

It is interesting to show cases where expectation formation for the output gap is anchored or less forward-looking as expectation formation for inflation rate. We show that we can observe similar outcomes as expectation formation for an inflation rate when the same expectation formation for the output gap is assumed. These outcomes can justify the role of government to escape from a liquidity trap.

6.1 Expectation Formation for the Output Gap

As we analyzed the effects of inflation expectation formation, now we can examine how expectation formation for the output gap affects monetary policy. We define expectation formation for the output gap in an analogous way to an inflation expectation as follows.

$$x_{t+1}^e := \begin{cases} \gamma_x \mathrm{E}_t x_{t+1} + (1-\gamma_x)\bar{x}, & \text{(a)} \\ \gamma_x \mathrm{E}_t x_{t+1} + (1-\gamma_x)x_t, & \text{(b)} \end{cases}$$

where γ_x is a parameter satisfying $0 \leq \gamma_x \leq 1$. The case (a) corresponds to the situation in which the output gap expectation is partially anchored at $\bar{x}$, where $\bar{x}$ denotes a certain level of output gap. When $\gamma_x = 0$, the output gap expectation is perfectly anchored. The case (b) corresponds to a situation in which the output gap expectation is given by a weighted average between forward-looking rational expectation and a current output gap. When $\gamma_x = 1$, x_{t+1}^e is reduced to perfectly forward-looking, i.e., $\mathrm{E}_t x_{t+1}$. The output gap expectation becomes less forward-looking as γ_x decreases. We set $\bar{x} = 0$ as a baseline calibration.

6.2 Simulations

6.2.1 Optimal Commitment Policy

Figure 11 shows impulse responses for different expectation formation for the output gap under optimal commitment policy. Basically, effects of monetary policy on an inflation rate and the output gap do not change greatly with different expectation formations under optimal commitment policy. In the case of a changing degree in anchoring expectation, as the expectation of the output gap is anchored more firmly, reductions of an inflation rate and the output gap more decrease in the early periods. In details, however, we can find some differences from the case of expectation formation for an inflation rate. The zero interest rate policy continues for the longest periods in the case of a perfectly anchored output gap expectation. The reason for this is given by smaller accumulated elasticity of the output gap to a real interest rate. We explain this observation with Figure 12. Figure 12 plots impulse responses by changing γ_x in $x^e_{t+1} = \gamma_x \mathrm{E}_t x_{t+1} + (1-\gamma_x)\bar{x}$. Periods of zero interest rate becomes longer as γ_x becomes lower. Simultaneously, reductions of an inflation rate and the output gap in early periods become larger as γ_x becomes lower, except in the case of $\gamma_x = 0$. This is because a power of forward guidance is weakened by anchoring the output gap expectation. Since we set $\bar{x} = 0$, equation (5) is similar to McKay, Nakamura, and Steinsson's (2016) discounted Euler equation in which there is a discounting parameter for the expected output gap. In a discounted Euler equation, the monetary policy loses the power to stimulate an economy since a future monetary easing has less effect on a current output gap.

Figure 11 shows that an inflation rate and the output gap respond differently under a case of low degree in forward-lookingness in forming output gap expectation. Responses of the nominal interest rate and real interest rate are almost identical as in a case of perfectly forward looking expectation for the output gap. The reduction of the output gap, however, is the largest in early periods in this case though such a reduction is sufficiently smaller in comparison to the case of the Taylor rule as shown in the following section.

6.2.2 The Taylor Rule

Under the Taylor rule given by equation (13), monetary policy effects sufficiently change according to expectation formation for the output gap. Figures 13 and 14 show the response of an economy when a degree in anchoring expectation for the output gap changes. The reduction of an inflation rate and the output gap at an initial period becomes larger as γ_x becomes higher. To avoid reductions in an inflation rate and the output gap, strong anchoring is needed for the output gap expectation. Note that the effects of changing γ_x is more sensitive to avoid a drop in the output gap in comparison to the effects of changing γ_π for the output gap as shown in Figure 5. Moreover, anchoring the output gap is also effective to mitigate a drop in an inflation rate. Thus, the output gap expectation formation can play an important role to escape from a liquidity trap at least as an inflation expectation formation can under the Taylor rule. In other words, the effects of monetary policy significantly change according to different inflation expectation formations under the Taylor rule.

When we look at the result of a low degree of forward-lookingness in output gap expectation formation, an inflation rate and the output gap decrease more as the degree of forward-looking is lower. Thus, in panels (a) and (b) in Figure 13, dotted lines decrease more than solid lines.

6.2.3 Simple Rule with Price-level Targeting

Figure 15 shows impulse responses under a simple rule with price-level targeting given by equation (14). The result is similar to the case of the Taylor rule. The inflation rate and the output gap change responding to the expectation formation for the output gap (Figure 15a, b). Monetary policy can avoid a large drop in an inflation rate and the output gap when the output gap expectation is strongly anchored. An inflation rate and the output gap decrease more as the degree of forward-lookingness is lower. However, reductions in an inflation rate and the output gap are not as large as those under the Taylor rule. This shows that a simple rule with price-level targeting is more effective than the Taylor rule in a liquidity trap under various expectation formation for the output gap.

7 Concluding Remarks

This paper shows that expectation formation has different outcomes for monetary policy in a liquidity trap. Such a difference is trivial for optimal monetary policy. However, for simple and realistic rules such as the Taylor rule and a rule with price-level targeting, we observe significant difference on the effect of monetary policy on economic dynamics according to expectation formation.

As well as anchoring an inflation expectation, anchoring the output gap expectation is effective to avoid large drops in an inflation rate and the output gap in a liquidity trap under simple rules. Therefore, both a central bank and government can play important roles to manage expectation to escape from a liquidity trap.

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SUPPLEMENTARY APPENDIX

A Additional Analysis

A.1 Welfare Losses

In this section, we show welfare loss and the rate of binding the ZLB. The unconditional welfare loss is as follows.

$$\mathrm{WL} = \frac{1}{2}\frac{\alpha\theta(1+\omega\theta)}{(1-\alpha)(1-\alpha\beta)}\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t(\pi_t^2+\lambda_x x_t^2).$$

We evaluate losses in terms of welfare equivalent consumption loss following Adam and Billi (2007)[15]:

$$p = 100\times\sigma\left[-1+\sqrt{\frac{2(1-\beta)\mathrm{WL}}{\sigma 100^2}}\right].$$

We simulate 1000 paths for 1000 quarters and average the discounted losses.

Table A1 shows results. Under optimal commitment policy, a difference in welfare loss is small for different expectation formations. A welfare loss is smallest when the expectation formation is completely rational. Under the Taylor rule, a difference in welfare loss is large for different expectation formations. Monetary policy achieves the smallest welfare loss when expectation formation is completely anchored. Under the simple price-level targeting rule, a difference in welfare loss is small except a case when a degree of forward-lookingness is small. A welfare loss is smallest when expectation formation is partly anchored.

A.2 Frequency of Binding the ZLB

Table A2 shows a frequency of binding the ZLB. The result shows that the welfare losses increase as the rate of binding increases.

[15] See Adam and Billi (2007, p.748).

A.3 Optimal Commitment Policy with Expectation Formation

We have analyzed optimal commitment policy by deriving first order conditions under rational expectation. This section derives first order conditions considering the expectation formation and shows numerical results.

First order conditions under optimal commitment policy when the inflation expectation is partly anchored (Case (a) of equation 1) are as follows:

$$\pi_t - \beta^{-1}\sigma\gamma_\pi\phi_{1t-1} - \sigma(1-\gamma_\pi)\phi_{1t} + \phi_{2t} - \gamma_\pi\phi_{2t-1} - \beta(1-\gamma_\pi)\phi_{2t} = 0,$$
$$\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0,$$
$$i_t\phi_{1t} = 0,\ \phi_{1t} \geq 0,\ i_t \geq 0.$$

When an agent reflects a current inflation to form an inflation expectation (Case (b) of equation 1), first order conditions under optimal commitment policy are as follows:

$$\pi_t - \beta^{-1}\sigma\gamma_\pi\phi_{1t-1} + \phi_{2t} - \gamma_\pi\phi_{2t-1} = 0,$$
$$\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0,$$
$$i_t\phi_{1t} = 0,\ \phi_{1t} \geq 0,\ i_t \geq 0.$$

Figure A1 depicts impulse responses. The result does not differ largely from Figure 3.

A.4 Low Elasticity of Intertemporal Substitution

This section shows numerical results when we set $\sigma = 2$ that is a closer value to that in Sugo and Ueda (2008).[16] See Figures A2 to A4.

B Numerical Algorithm

We solve the central bank's optimization problem by calculating the solution for equations (5) to (6) and equations (10) to (12). Since the zero lower bound (ZLB) introduces

[16]The estimated value by Sugo and Ueda (2008) is $\sigma = 0.8006$.

nonlinearity in the model, we employ a numerical technique which approximates expected variables.

First of all, we specify the grids for four state variables, r_t^n, ϕ_{1t-1}, and ϕ_{2t-1}. Let $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$ denote the vector of grids for r_t^n, ϕ_{1t-1}, and ϕ_{2t-1}, respectively. A tensor of these grid vectors, defined as $\mathbf{S} \equiv \mathbf{S}_1 \otimes \mathbf{S}_2 \otimes \mathbf{S}_3$, determines the combination of all grids. The size of $\mathbf{S}$ is $N = n_1 \times n_2 \times n_3 = 9261$. As for $\mathbf{S}_1$, we put relatively larger number of grids near the kink point stemming from the ZLB with the aim of mitigating the expected approximation error. The *p.d.f.* for the natural interest rate is discretized by Gaussian Quadrature.

Notice that we can rewrite the complementarity conditions regarding the ZLB, equations (12), as

$$\min(\max(\sigma\phi_{1t}, -i_t), \infty) = 0. \tag{A.1}$$

In order to employ an algorithmic solution that is designed basically for differentiable functions, we approximate equation (A.1) by a semismooth function, in a so-called Fischer's equation:

$$\psi^-(\psi^+(\sigma\phi_{1t}, -i_t), \infty) = 0,$$

where $\psi^{\pm}(u, v) = u + v \pm \sqrt{u^2 + v^2}$ (c.f., Miranda and Fackler, 2004).

Let $\mathbf{h}_t \equiv (x_t, \pi_t, \phi_{2t})$ denote the vector of forward-looking variables at time t. We need to obtain $\mathbf{h}_t$, i_t, and ϕ_{1t} by solving the central bank's optimization problem, taking state variables as given. In order to calculate the expectations terms, we approximate the time-invariant function for forward-looking variables, $\mathbf{h}$, by a collocation method. Our solution procedure is summarized as follows:

1. Given a particular set of grids for state variables, denoted by $\mathbf{S}^j$, and the initial guess of the functional form for $\mathbf{h}(\mathbf{S}^j)$, denoted by $\mathbf{h}^0(\mathbf{S}^j)$, compute $\mathbf{h}^1(\mathbf{S}^j)$, i_t, and $\phi_{1,t}$ as a solution for equations (5) to (6) and equations (10) to (12). A cubic-spline function is used to interpolate $\mathbf{h}(\mathbf{S^j})$.

2. Repeat step 1 for all $j = 1, \ldots N$.

3. Stop if $\|\mathbf{h}^1 - \mathbf{h}^0\|_\infty / \|\mathbf{h}^0\|_\infty < 1.5 \times 10^{-6}$. Otherwise, update the initial functional form as $\mathbf{h}^0 \equiv \mathbf{h}^1$ and go to step 1.

Euler residuals from first order conditions are in the order of 10^{-3}, which is concentrated mostly around the zero lower bound. Computation time is around 4 hours. The software used is Matlab, CPU is Xeon with 3.60GHz, and Memory is 32GB.

Table 1: The degree of how much expectation is anchored: Case (a)

$$\pi^e_{t,t+4} = \gamma_\pi \mathrm{E}_t \pi_{t,t+4} + (1-\gamma_\pi)\bar{\pi}$$

$$\pi_{t,t+4} - \pi^e_{t,t+4} = \beta^A(\pi^e_{t,t+4} - 1\%) \times D + \beta^B(\pi^e_{t,t+4} - 2\%) \times (1-D) + \varepsilon_{t,t+4}$$

	β^A	γ^A_π	β^B	γ^B_π	Observations
Equation (3)	0.060 (0.081)	0.944	0.556* (0.300)	0.643	90

Note: The data on inflation forecasts is obtained from Consensus Economics and covers from June 1994:Q1 to 2016:Q2. We use core inflation rates for π. $\bar{\pi}$ is set to be 1% before the introduction of an inflation target and set to be 2% after that. D takes one before 2013, otherwise zero. Standard errors in parentheses are calculated by the Newey-West (1987) estimator. Here, ***, **, and * indicate 1%, 5%, and 10% significance, respectively.

Table 2: The degree of how much expectation formation is forward-looking: Case (b)

$$\pi^e_{t,t+4} = \gamma_\pi \mathrm{E}_t \pi_{t,t+4} + (1-\gamma_\pi)\pi_{t-4,t}$$

$$\pi_{t,t+4} - \pi^e_{t,t+4} = \beta(\pi^e_{t,t+4} - \pi_{t-4,t}) + \varepsilon_{t,t+4}$$

	β	γ_π	Observations
Equation (4)	0.246^{*} (0.130)	0.803	90

Note: The data on inflation forecasts is obtained from Consensus Economics and covers from June 1994:Q1 to 2016:Q2. We use core inflation rates for π. Standard errors in parentheses are calculated by the Newey-West (1987) estimator. Here, ***, **, and * indicate 1%, 5%, and 10% significance, respectively.

Table 3: Parameter values

Parameters	Values	Explanation
β	0.995	Discount Factor
σ	6.25	Elasticity of Output Gap to Real Interest Rate
α	0.875	Price Stickiness
θ	6	Elasticity of Goods Demand
ω	2.149	Elasticity of Marginal Cost
i^*	0.5	Steady State Nominal Interest Rate
σ_r	0.102	Standard Deviation of Natural Rate Shock
ρ_r	0.892	Persistence of Natural Rate Shock
ψ_π	1.5	Coefficient of Inflation Rate in Taylor Rule
ψ_x	0.5	Coefficient of Output Gap in Taylor Rule
ψ_p	1.5	Coefficient of Price Level in Price-level Targeting Rule
$\bar{\pi}$	0	Anchored Level of Inflation Rate
$\bar{x}$	0	Anchored Level of Output Gap
ρ_i	0.5	Interest Rate Smoothing

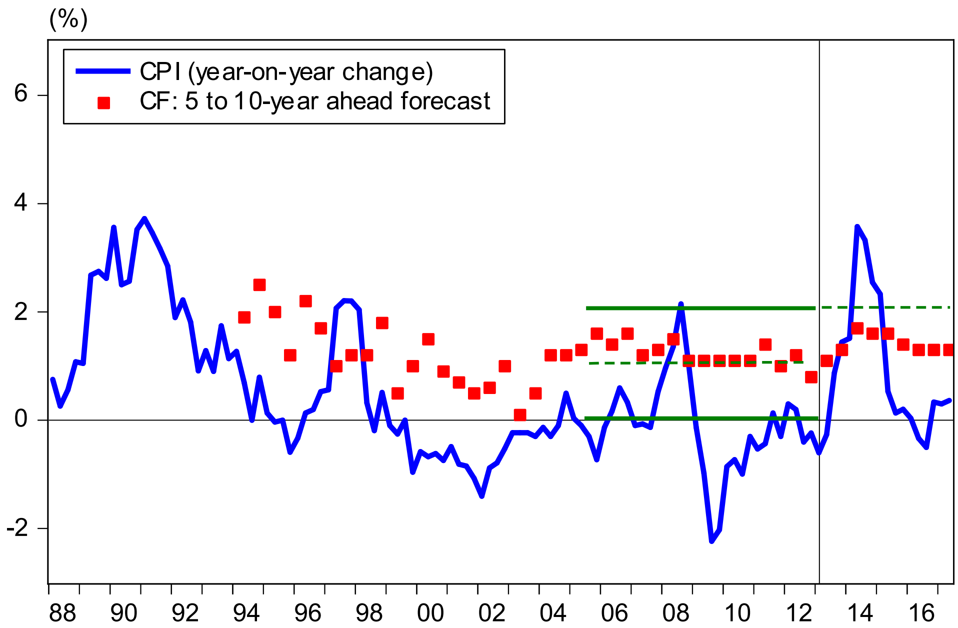

Figure 1: Longer-term inflation forecasts (5 to 10-year ahead forecasts of Consensus Forecasts from Consensus Economics) and inflation rates in Japan. The solid and dashed lines are defined as upper and lower limits of inflation targets and the point targets or the middle points, respectively. The vertical lines show the time when inflation targets are introduced.

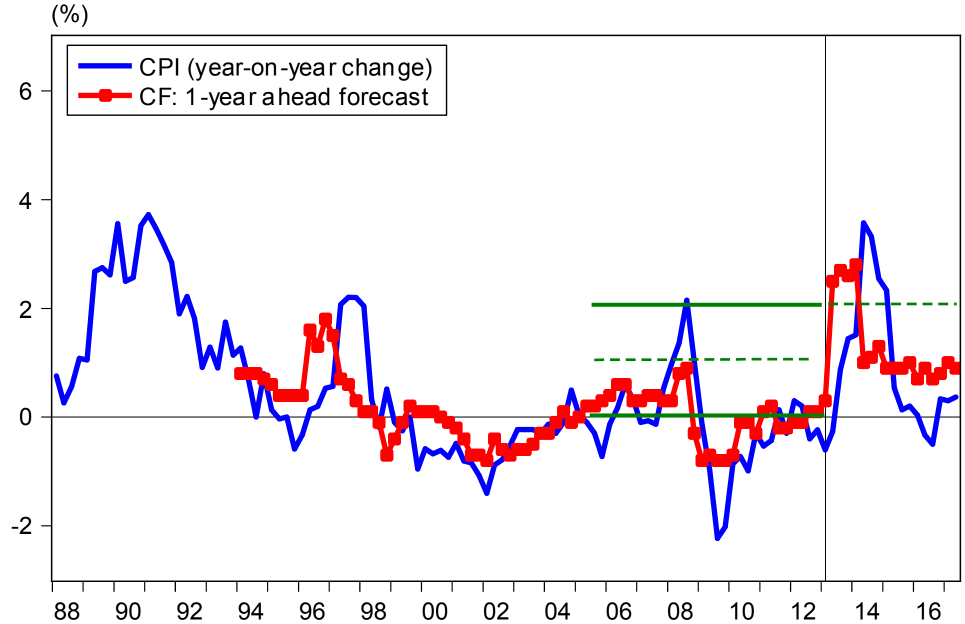

Figure 2: Short-term inflation forecasts (1-year ahead forecasts of Consensus Forecasts from Consensus Economics) and inflation rates in Japan. The solid and dashed lines are defined as upper and lower limits of inflation targets and the point targets or the middle points, respectively. The vertical lines show the time when inflation targets are introduced.

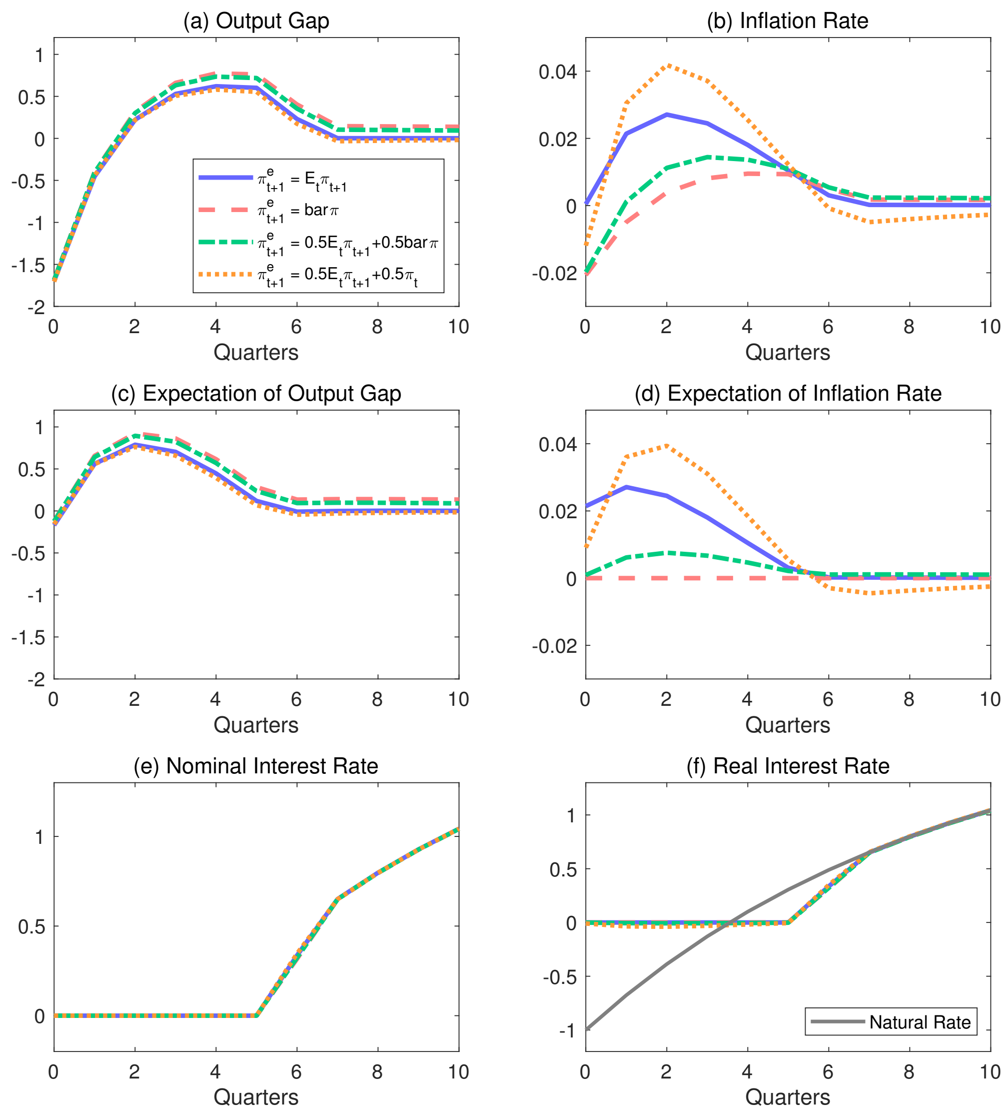

Figure 3: Impulse responses to an annual −3% natural rate shock under optimal commitment policy for different expectation formations for an inflation rate.

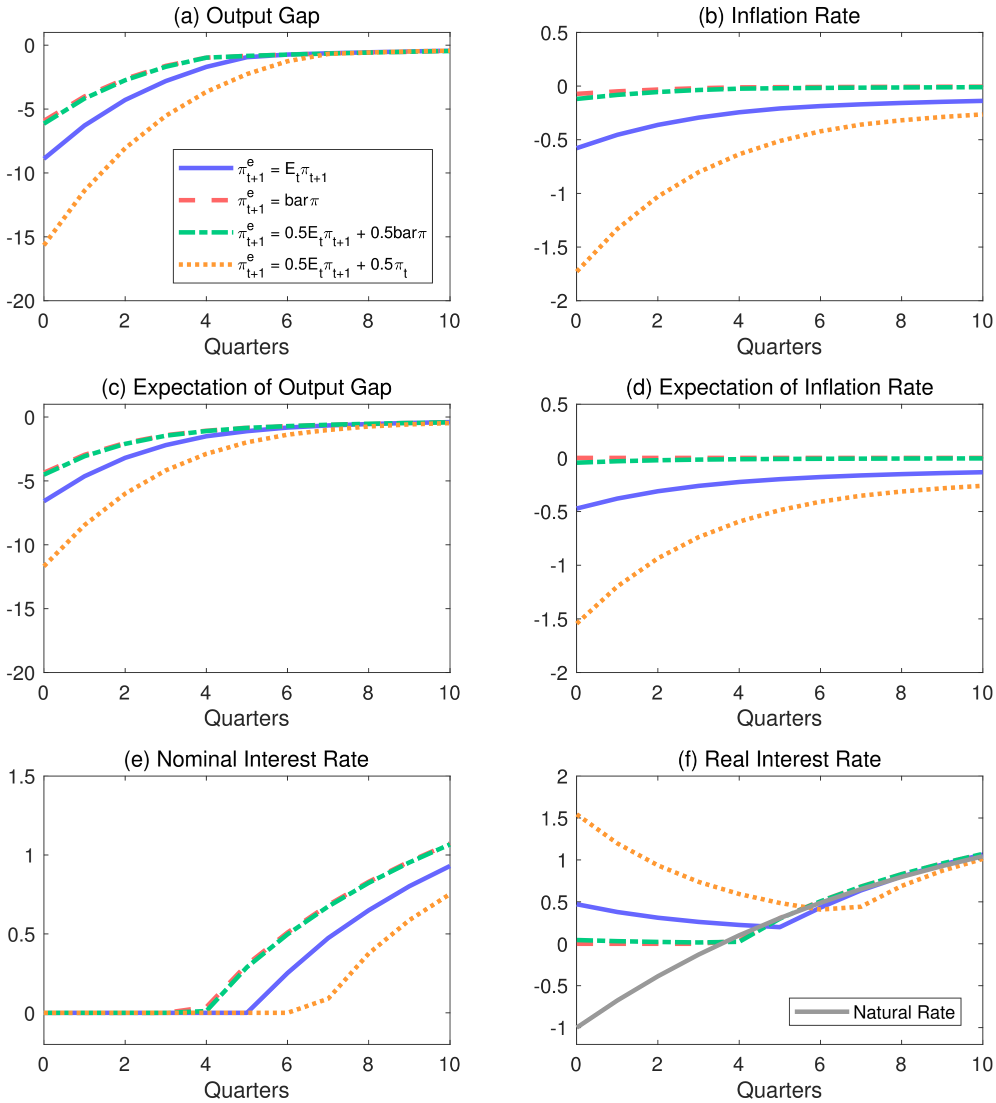

Figure 4: Impulse responses to an annual -3% natural rate shock under the Taylor Rule for different expectation formations for an inflation rate.

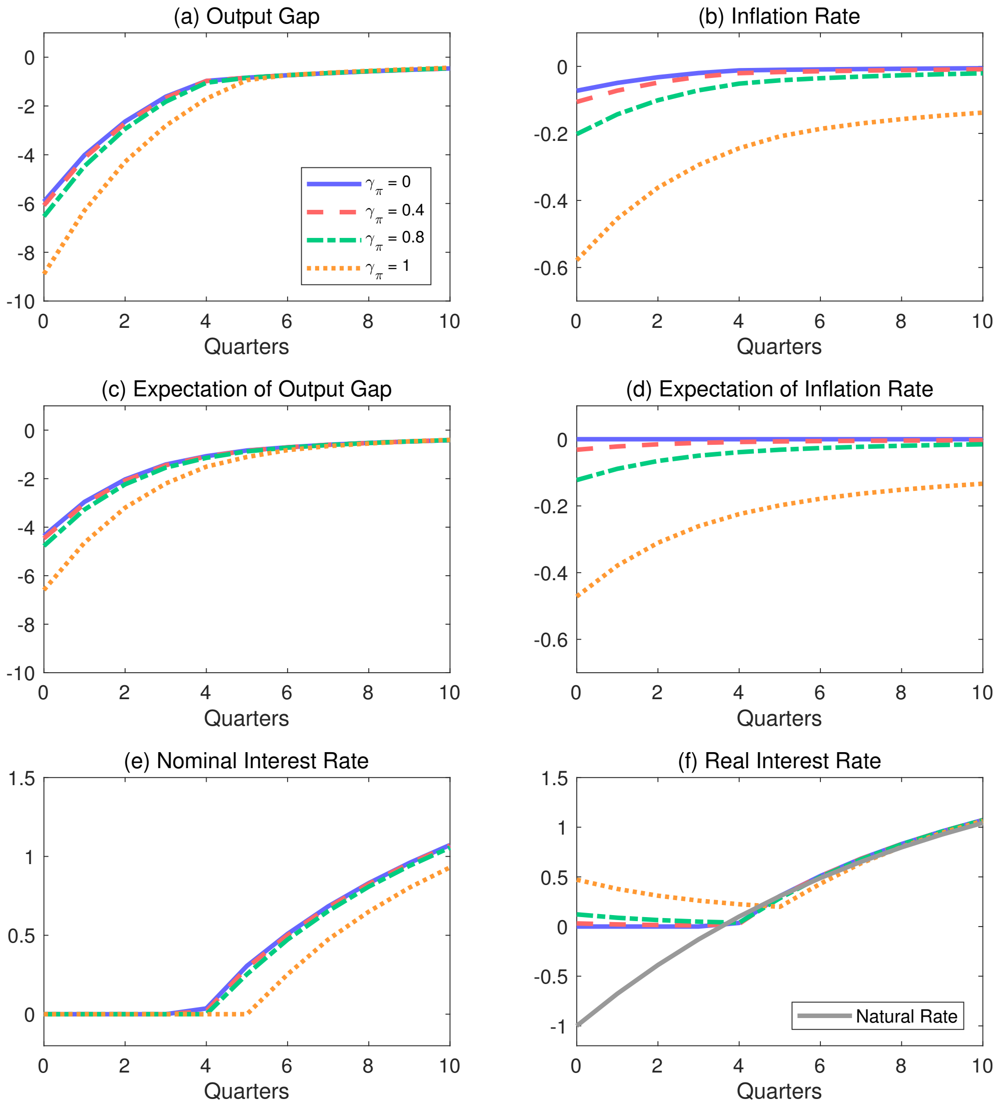

Figure 5: Impulse responses to an annual -3% natural rate shock for various values of γ_π in $\pi^e_{t+1} = \gamma_\pi \mathrm{E}_t \pi_{t+1} + (1 - \gamma_\pi)\bar{\pi}$ under the Taylor Rule.

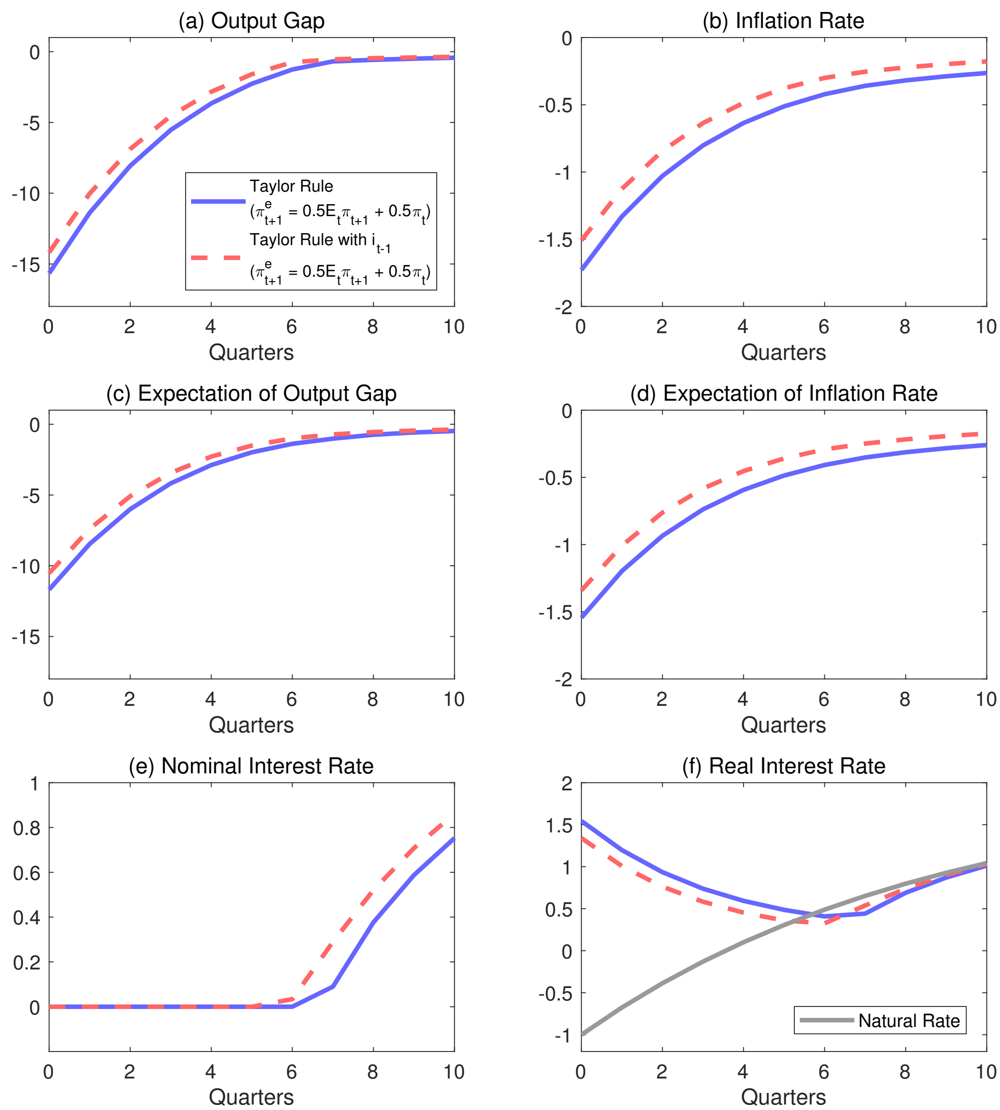

Figure 6: Impulse responses to an annual -3% natural rate shock under the Taylor Rule with and without interest rate smoothing.

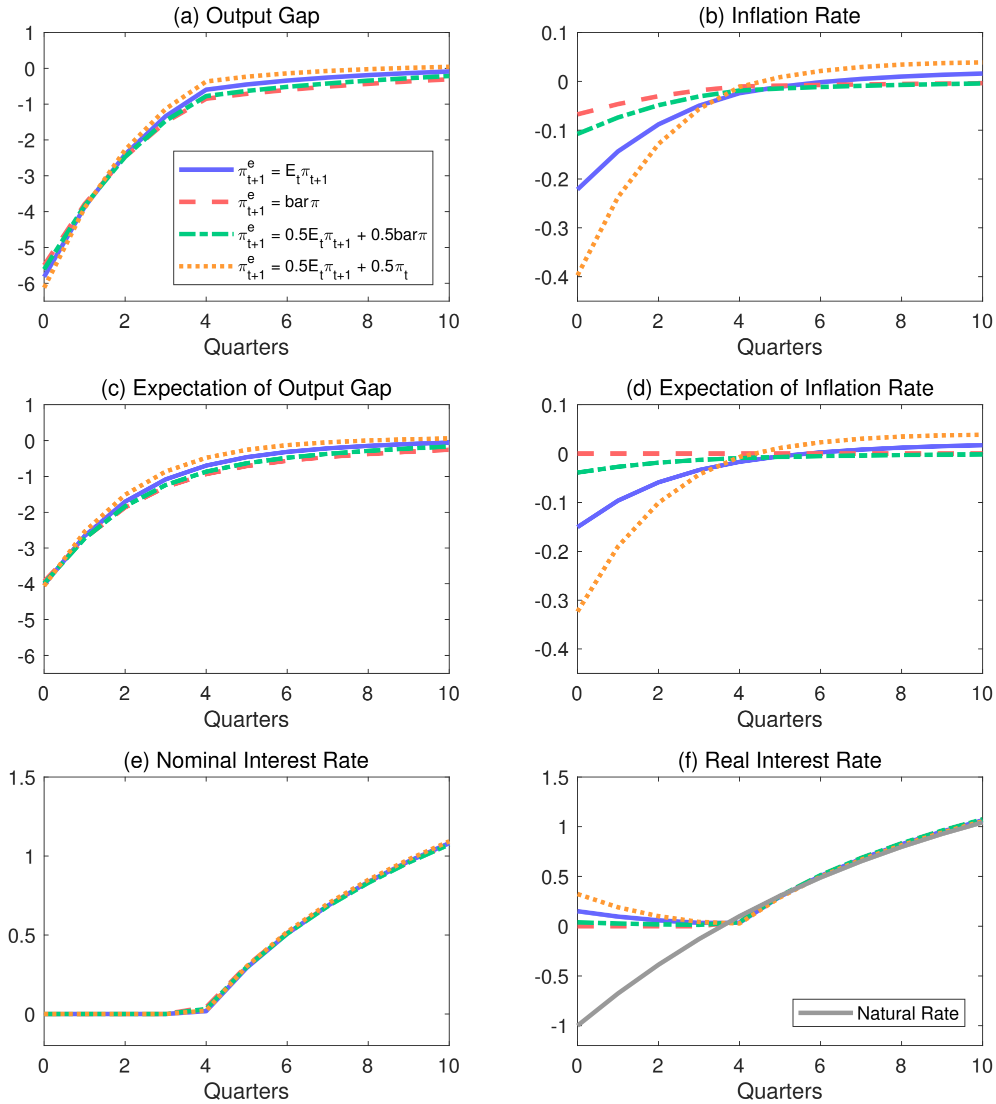

Figure 7: Impulse responses to an annual -3% natural rate shock under a simple rule with price-level targeting for different expectation formations for an inflation rate.

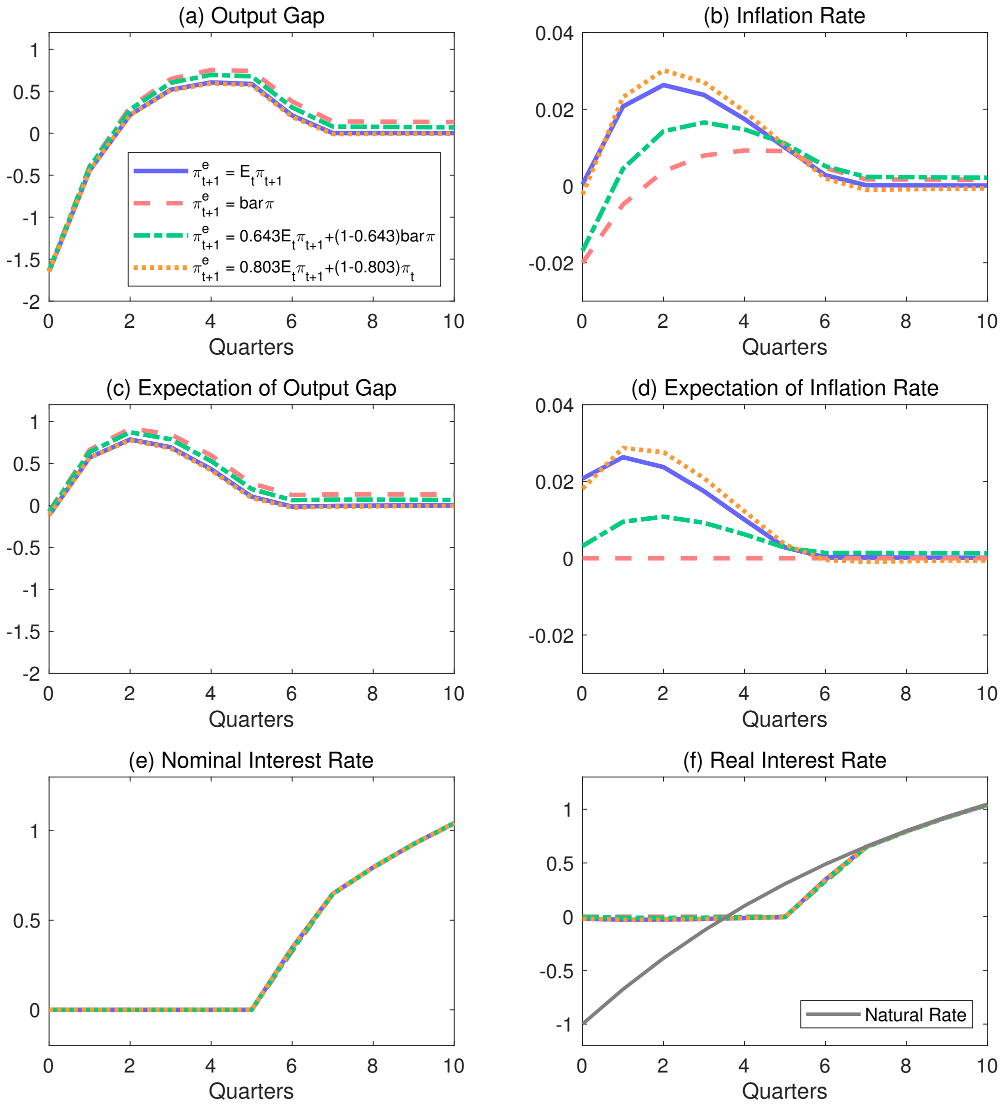

Figure 8: Impulse responses to an annual -3% natural rate shock under commitment with estimated value of γ_π for the Japanese economy.

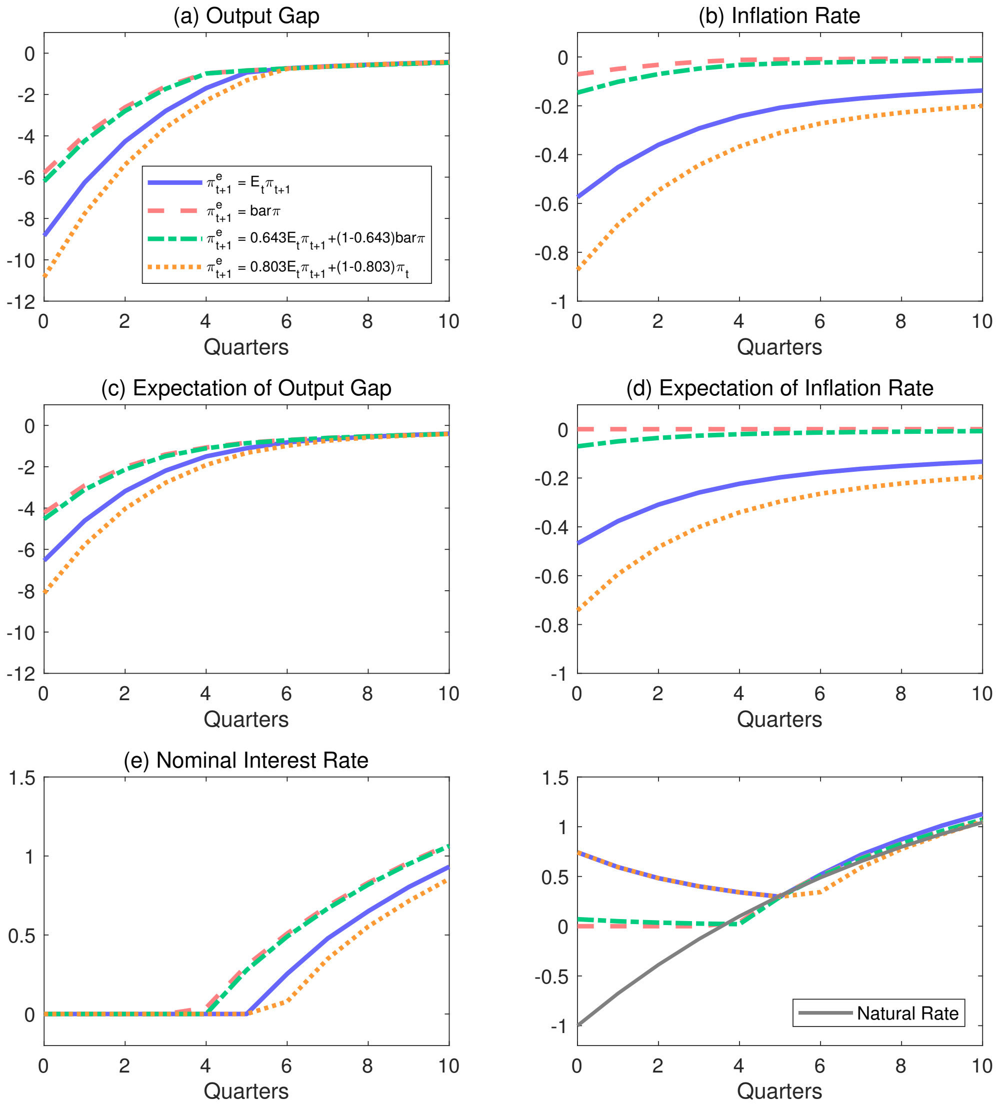

Figure 9: Impulse responses to an annual -3% natural rate shock under the Taylor rule with estimated value of γ_π for the Japanese economy.

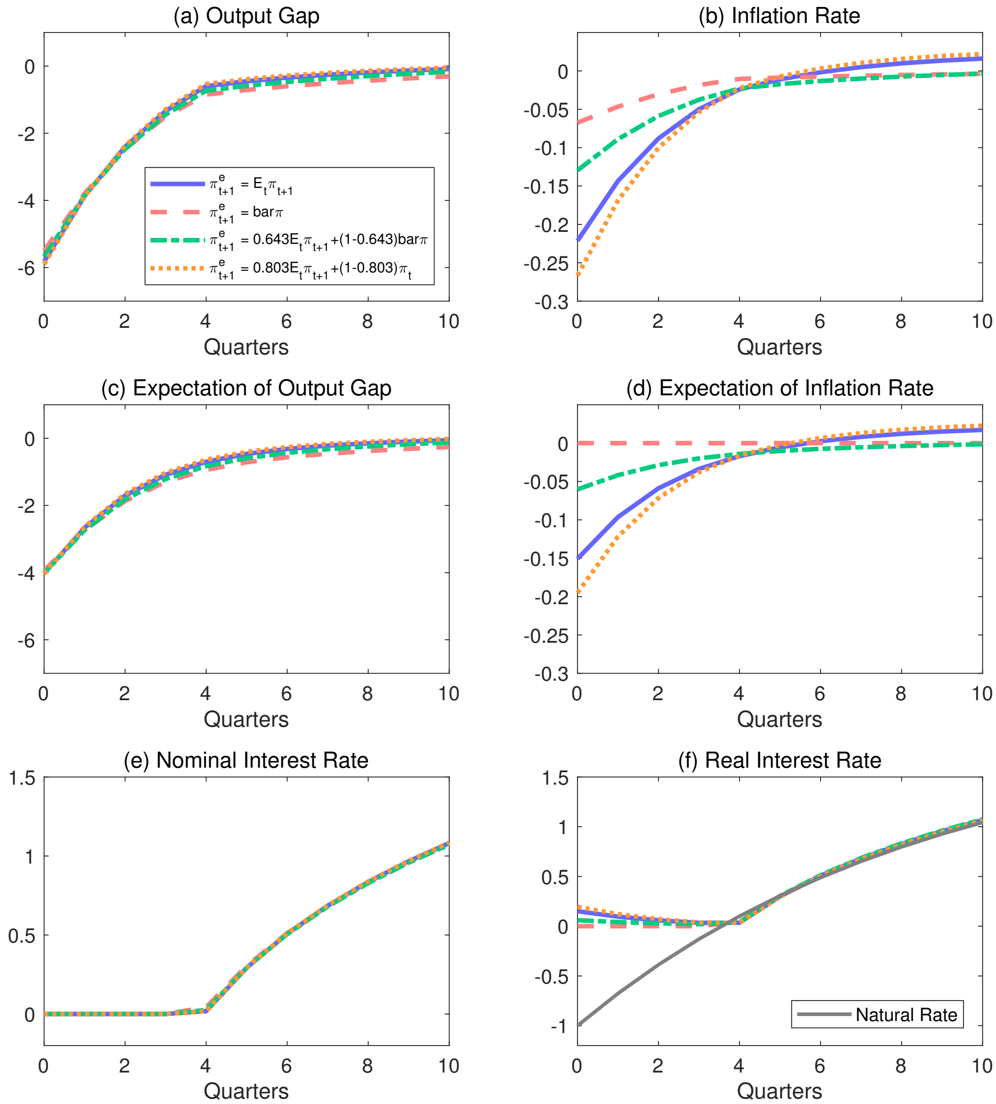

Figure 10: Impulse responses to an annual -3% natural rate shock under a simple rule with price-level targeting with estimated value of γ_π for the Japanese economy.

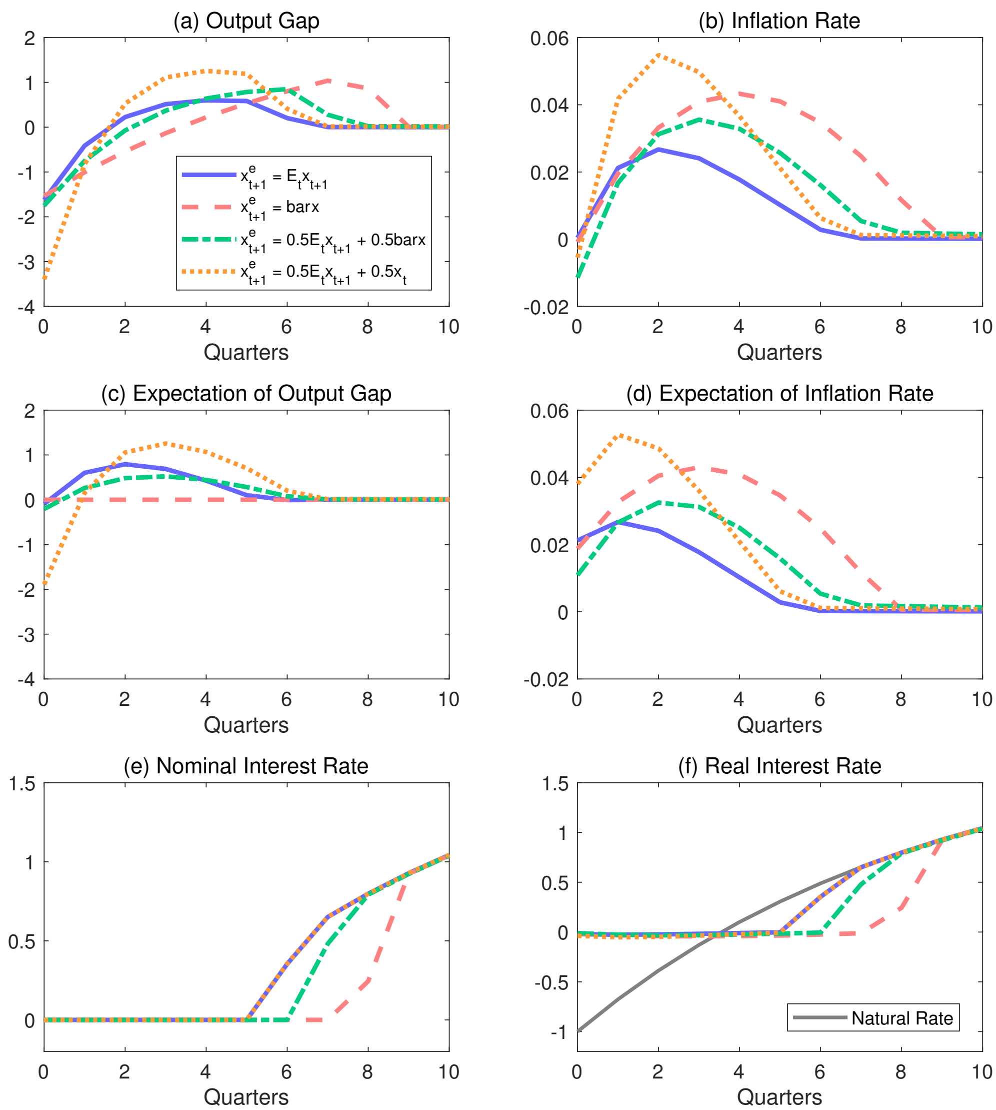

Figure 11: Impulse responses to an annual -3% natural rate shock under optimal commitment policy for different expectation formations for the output gap.

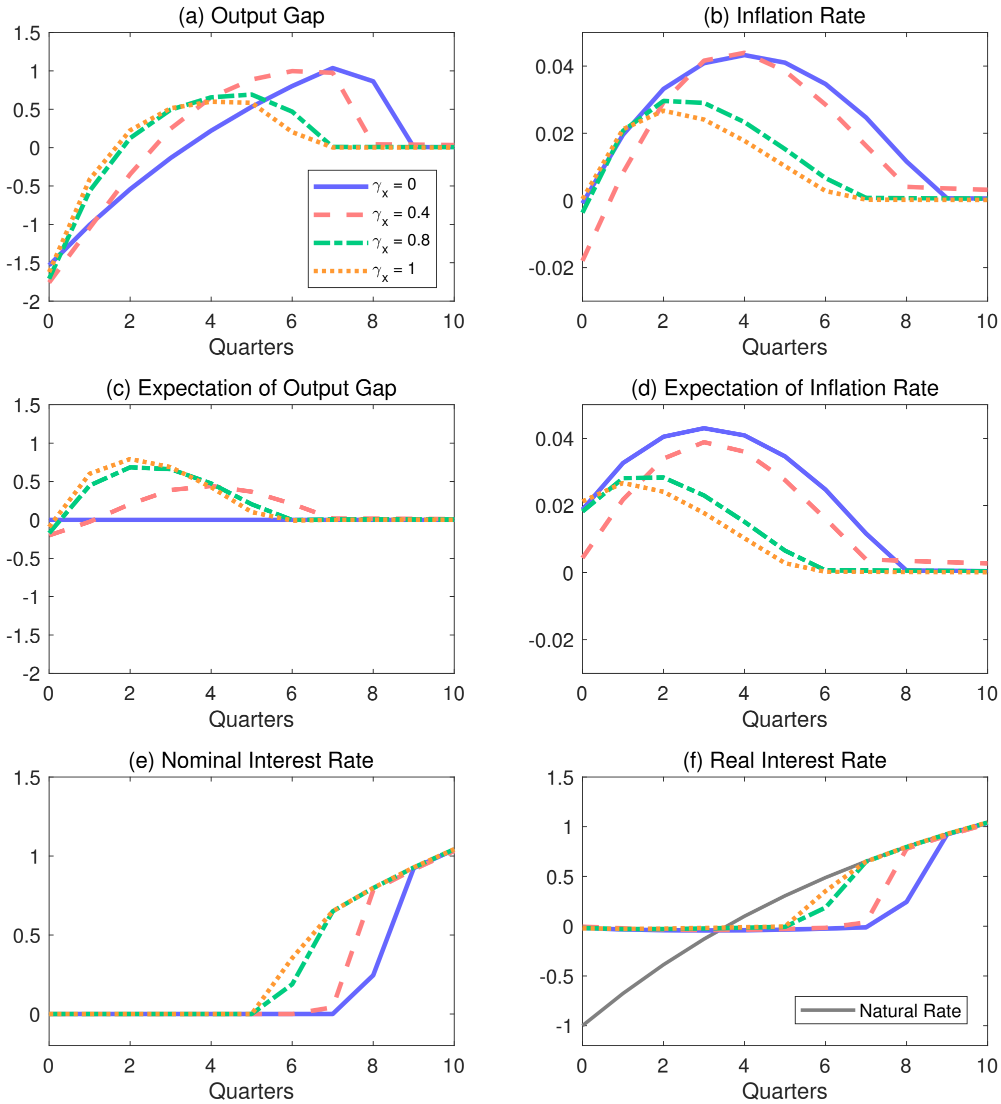

Figure 12: Impulse responses to an annual -3% natural rate shock for various values of γ_x in $\gamma_x \mathrm{E}_t x_{t+1} + (1-\gamma_x)\bar{x}$ under optimal commitment policy.

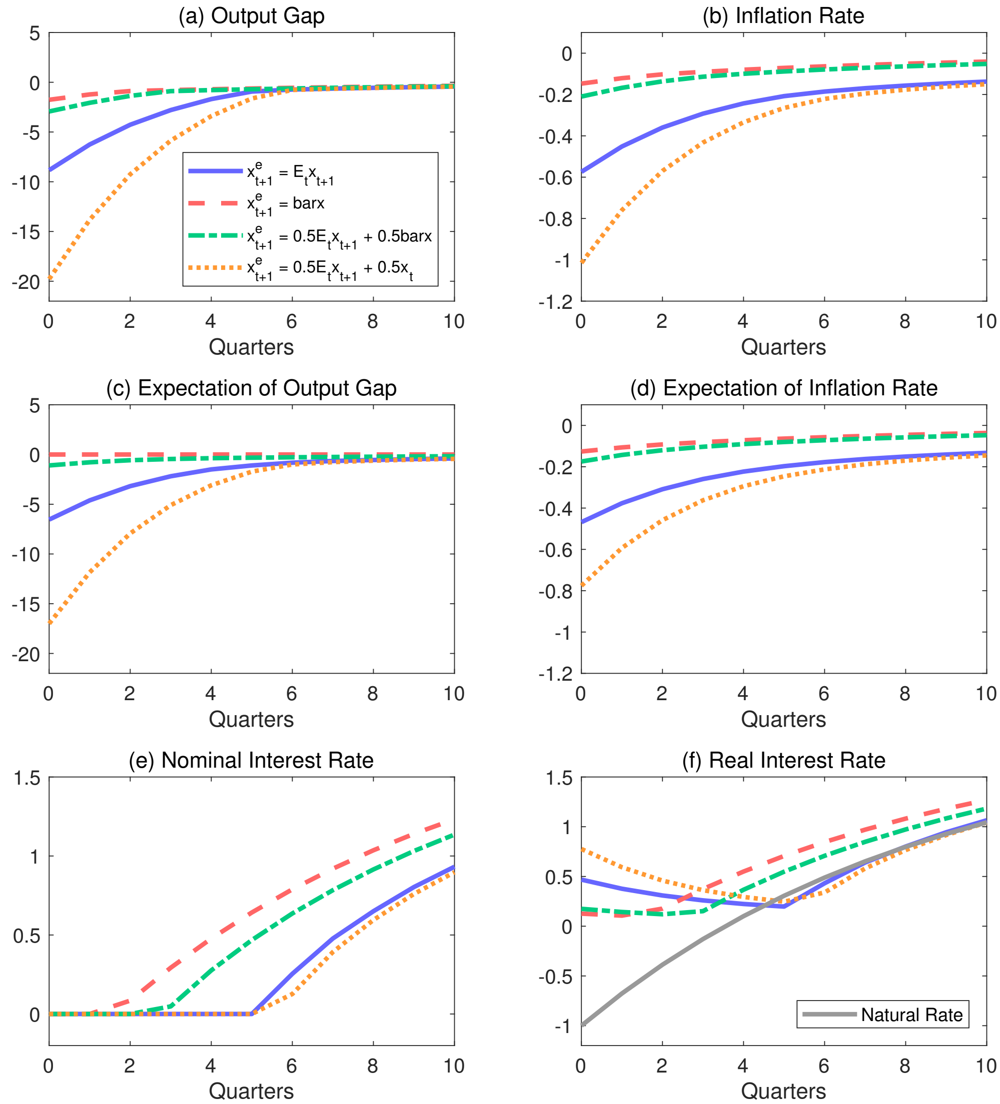

Figure 13: Impulse responses to an annual −3% natural rate shock under the Taylor rule for different expectation formations for the output gap.

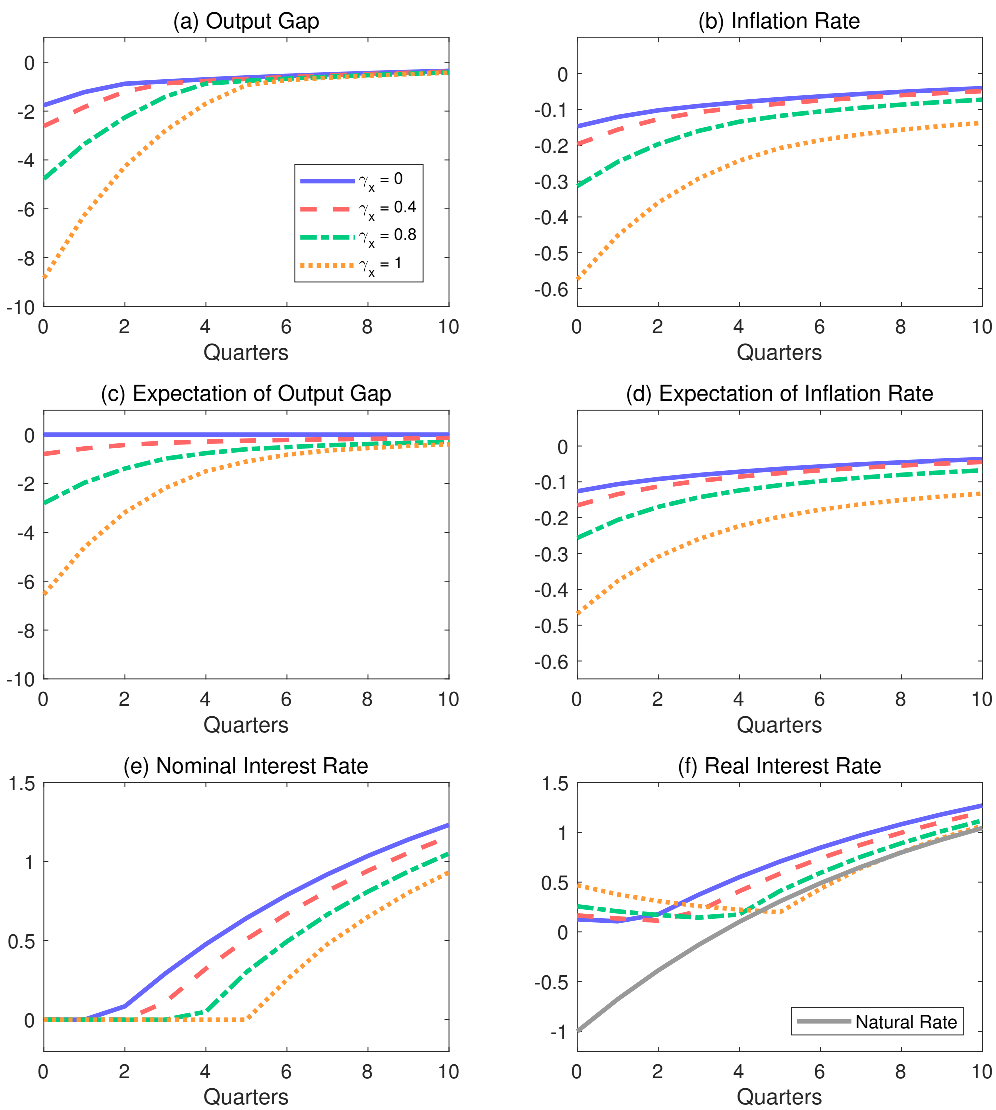

Figure 14: Impulse responses to an annual -3% natural rate shock for various values of γ_x in $\gamma_x \mathrm{E}_t x_{t+1} + (1-\gamma_x)\bar{x}$ under the Taylor rule.

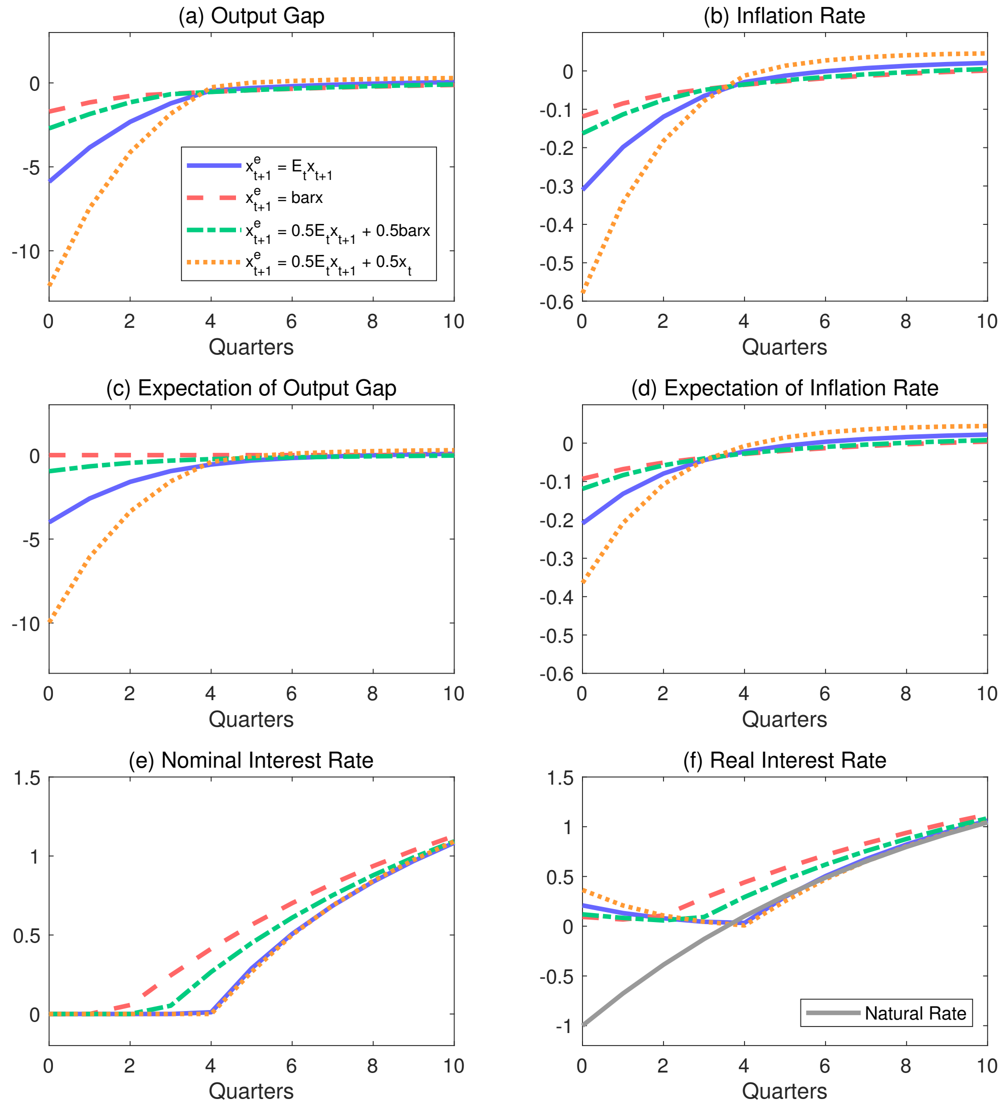

Figure 15: Impulse responses to an annual -3% natural rate shock under a simple rule with price-level targeting for different expectation formations for the output gap.

π^e_{t+1}	Commitment	Taylor Rule	Price-level Rule
$\mathrm{E}_t\pi_{t+1}$	5.4692×10^{-5}	0.020314	0.002996
$\bar{\pi}$	7.4898×10^{-5}	0.003312	0.002973
$0.5\mathrm{E}_t\pi_{t+1} + 0.5\bar{\pi}$	6.4395×10^{-5}	0.003529	0.002946
$0.5\mathrm{E}_t\pi_{t+1} + 0.5\pi_t$	5.7112×10^{-5}	0.080471	0.003807

Table A1: Welfare equivalent consumption losses

π^e_{t+1}	Commitment	Taylor Rule	Price-level Rule
$\mathrm{E}_t\pi_{t+1}$	1.044%	3.370%	2.220%
$\bar{\pi}$	1.049%	1.726%	2.094%
$0.5\mathrm{E}_t\pi_{t+1} + 0.5\bar{\pi}$	1.048%	1.849%	2.136%
$0.5\mathrm{E}_t\pi_{t+1} + 0.5\pi_t$	1.042%	6.402%	2.242%

Table A2: Frequency of binding the ZLB

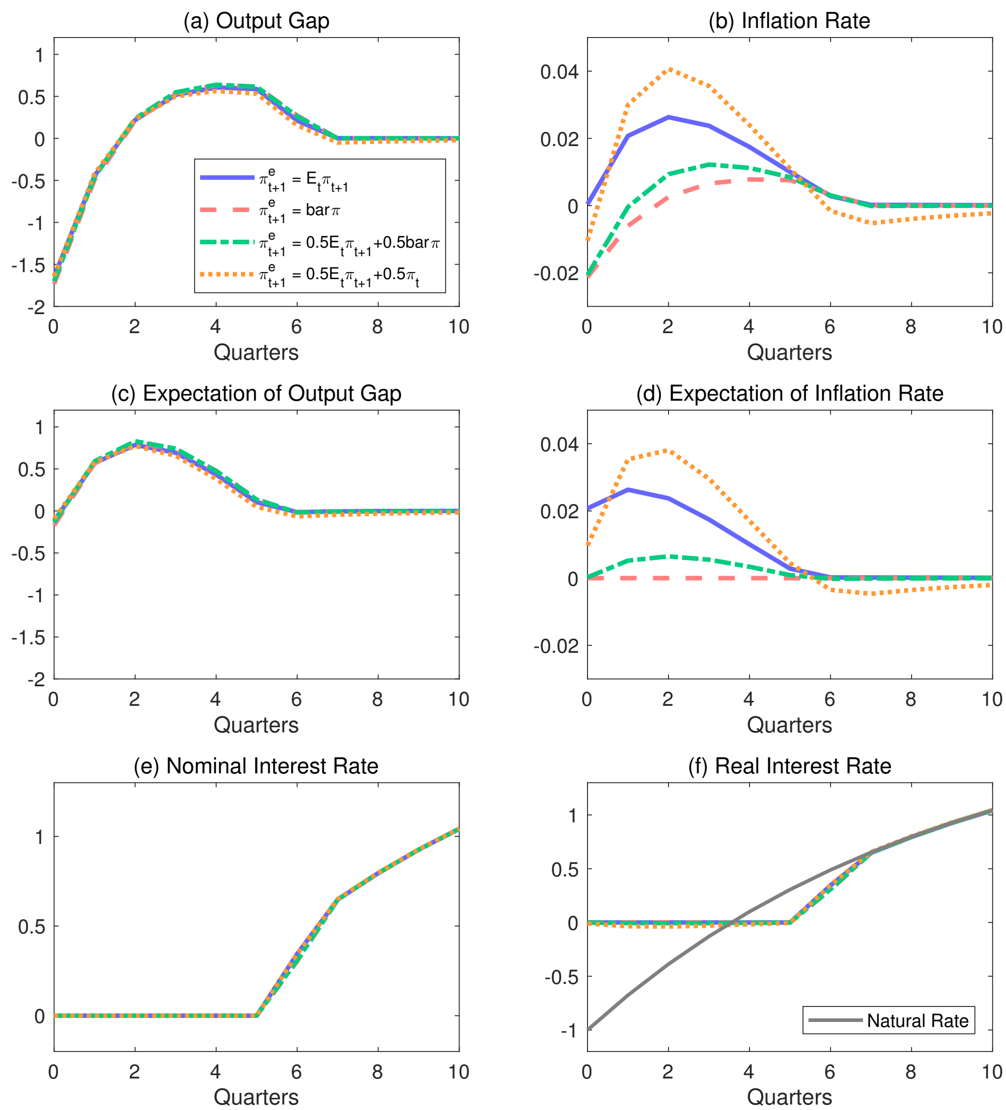

Figure A1: Impulse responses to an annual -3% natural rate shock under optimal commitment policy for different expectation formations for an inflation rate. First order conditions are derived under adaptive expectation and anchored expectation.

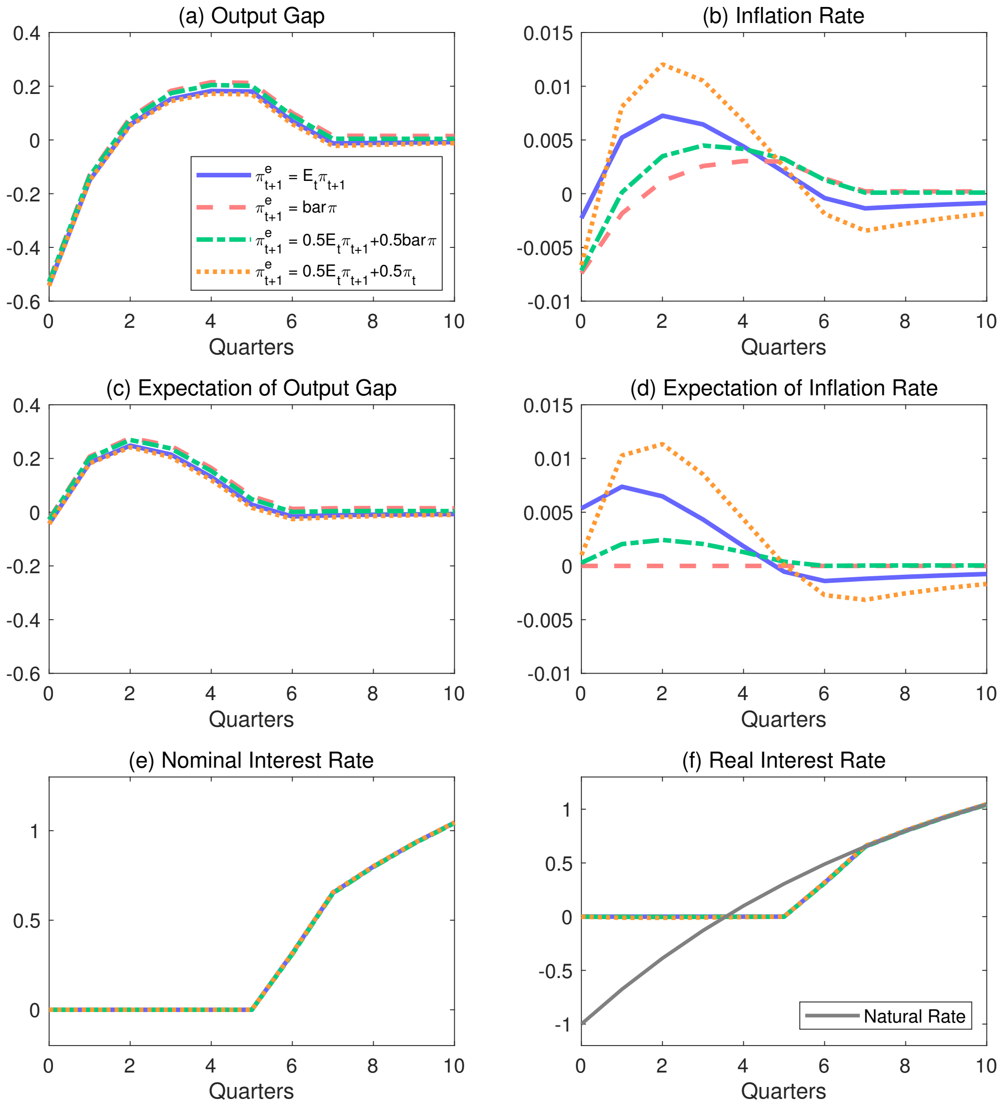

Figure A2: Impulse responses to an annual −3% natural rate shock under optimal commitment policy for different expectation formations for an inflation rate.

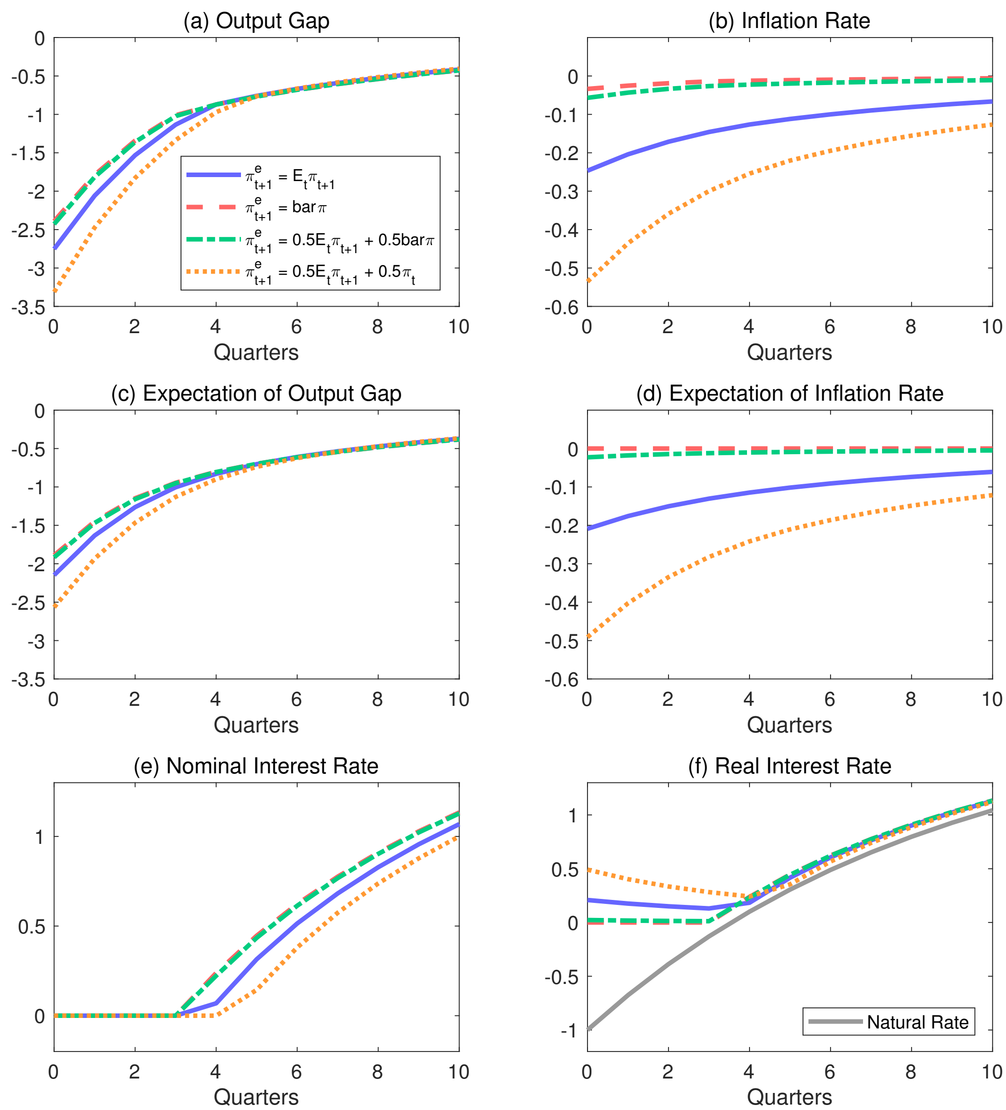

Figure A3: Impulse responses to an annual −3% natural rate shock under the Taylor Rule for different expectation formations for an inflation rate.

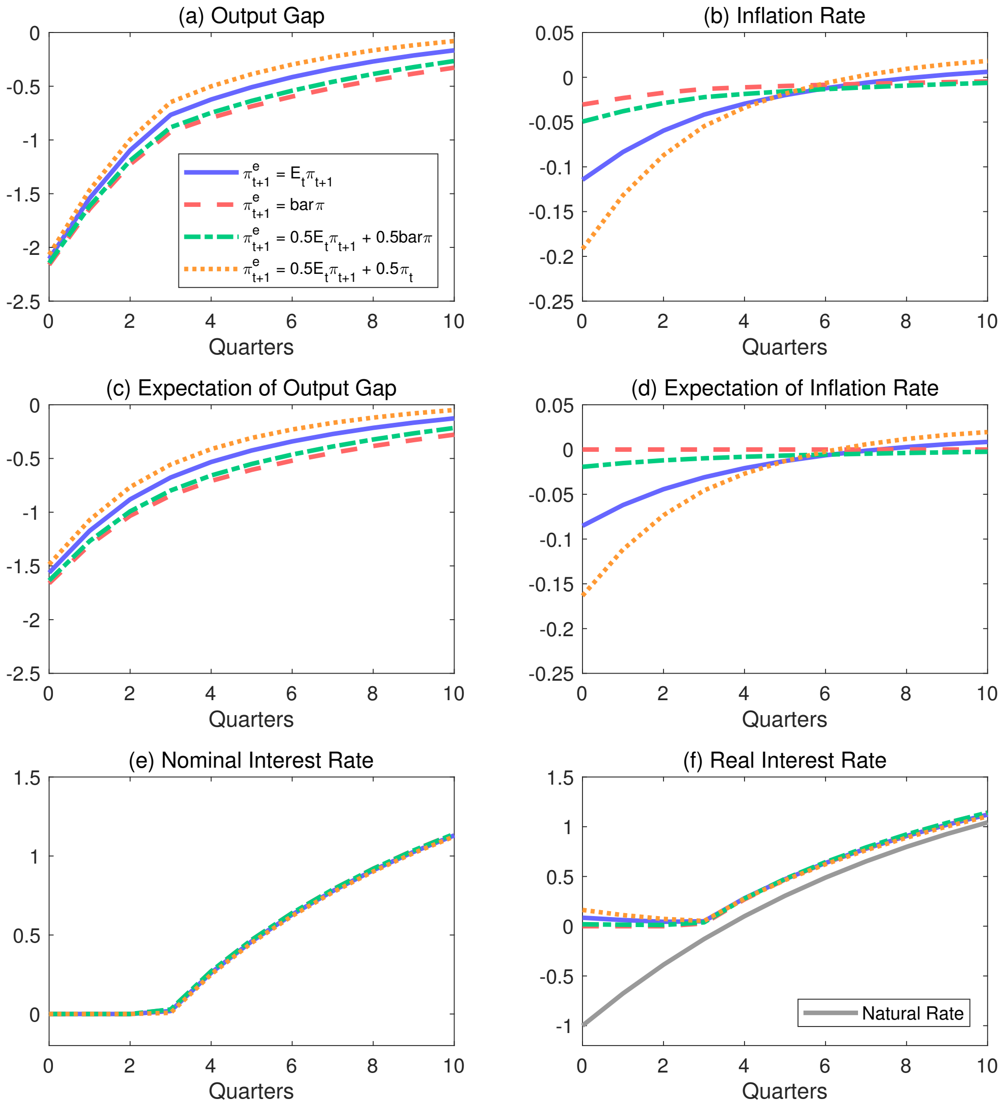

Figure A4: Impulse responses to an annual −3% natural rate shock under a simple rule with price-level targeting for different expectation formations for an inflation rate.