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Abstract

This paper analyzes the distinction between payment instruments and collateral in the interbank payment system. Given the interbank market is an over-the-counter market, decentralized settlement of bank transfers is inefficient if bank loans are illiquid. In this case, a collateralized interbank settlement contract improves efficiency through a liquidity-saving effect. The large value payment system operated by the central bank can be regarded as an implicit implementation of such a contract. This result explains why banks swap Treasury securities for bank reserves despite that both are liquid assets. This paper also discusses if a private clearing house can implement the contract.

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1 Introduction

Base money consists of cash and bank reserves. The latter type of money is used by banks when they settle bank transfers between their depositors. Typically, the daily transfer of bank reserves in a country is as large as a sizable fraction of annual GDP. This large figure implies that banks do not hold bank reserves merely to satisfy a reserve requirement, but also to settle the transfer of deposit liabilities due to daily bank transfers. In fact, several countries have abandoned a reserve requirement.[1] Banks in these countries still settle bank transfers through a transfer of bank reserves.

But why do banks need bank reserves for interbank payment? Theoretically, banks should be able to pay Treasury securities, i.e., the other liquid liabilities issued by the consolidated government. Unlike payers in retail payment, banks can easily handle these wholesale assets. Also, banks obtain bank reserves in exchange for Treasury securities through open market operations. Why do banks swap liquid assets for liquid assets?

Using a model of an interbank payment system, this paper explains this observation by the fact that the interbank market is an over-the-counter (OTC) market. The model involves three key assumptions: a bank must pay a penalty if it fails to send bank transfers requested by its depositors by a deadline; banks need payment instruments to settle bank transfers due to limited commitment; and banks negotiate the terms of each transaction between them bilaterally. The last assumption is the feature of an OTC market.

In this environment, there occurs a hold-up problem if interbank settlement is decentralized in an OTC market. In this case, a bank receiving a bank transfer can require the originating bank to pay a higher value of assets than the face value of the bank transfer, because it can threaten the originating bank with the penalty in case of a failure to send a bank transfer. The presence of this premium reduces the efficiency of the payment system by increasing the amount of liquidity necessary for interbank settlement.

For this problem, introducing an interbank settlement contract has a liquidity-saving effect. In this contract, banks submit collateral to a custodian first, and then the custodian transfers the balance of collateral between banks according to bank-transfer requests reported by each bank. This con-

[1] These countries include Australia, Canada, Denmark, Mexico, New Zealand, Norway, Sweden, and the U.K.

tract obviates the need for ex-post bargaining to settle bank transfers, because the amount of collateral to be transferred is determined by a contingent rule specified in the contract. This effect of the contract eliminates a premium due to a hold-up problem in decentralized interbank settlement.

The interbank payment system operated by the central bank, so-called a large value payment system, can be seen as an implicit implementation of such a contract, with the central bank being the custodian of collateral. This interpretation is based on the fact that banks can settle an outgoing bank transfer unilaterally by sending the same nominal value of bank reserves in the system. Also, the role of collateral in the contract is consistent with the fact that banks obtain bank reserves in exchange for their assets through open market operations. In the model, the collateral must be liquid due to an assumption that the central bank cannot handle illiquid assets. Thus, banks swap liquid collateral for bank reserves.

The model also shows that the custodian of collateral can implement this contract as an implicit contract just by holding submitted collateral until the end of a settlement cycle. Thus, it does not have to confiscate collateral in any event. This result is consistent with the fact that banks participate into the large value payment system voluntarily and are not bound to stay in the system by any explicit contract.

At the end, this paper discusses whether the custodian of collateral has to be the central bank. This question is motivated by recent developments in private large value payment systems, such as the Clearing House Interbank Payment System (CHIPS) in the U.S. and the CLS for foreign exchange settlement.[2] These observations have made the role of the central bank in interbank payment debatable.[3] The model shows that after the settlement of bank transfers, the custodian of collateral must return the remaining balance of collateral to each bank. The transfer of collateral is unilateral; the custodian must release collateral in return for nothing. This result is consistent with the fact that the central bank absorbs bank reserves through open market operations despite that bank reserves are worthless for the central bank.

Thus, while collateral makes the interbank settlement contract robust to limited commitment by banks, the custodian of collateral must be able to

[2]For the function of the CLS, see Kahn and Roberds (2001).

[3]Even if the central bank were not involved with the interbank payment system, it would still anchor the entire payment system as the supplier of central-bank notes, i.e., legal tender.

commit to not walking away with collateral. If a private clearing house has the same commitment ability as the central bank, then it can take over the role of the central bank in interbank payment. Otherwise, it needs some commitment device. In this regard, bank reserves are ideal collateral for a private clearing house, as they cannot be taken out of the central bank's large value payment system. Thus, a private clearing house might still need the central bank as the custodian of collateral, even if it provided a large value payment system on behalf of the central bank. This implication of the model is consistent with the current observation that the CHIPS and the CLS use bank reserves as collateral.

1.1 Related literature

This paper adds to the literature on the role of collateral in the large value payment system. Kahn (2013) analyzes the competition between a public and a private system, and shows that the cost of collateral necessary to make payments is critical for the endogenous choice of the payment system. Given this result, he demonstrates that the presence of a private large value payment system limits the central bank's ability to manipulate monetary policy. Kahn (2009) analyzes this issue in the context of cross-border settlement with a time difference. Also, Kahn and Roberds (2009) model the vertical integration of a public and a private system through tiering, and show the collateral-saving effect of tiering in the payment system. In these papers, payment instruments and collateral are distinguished by an exogenous collateral constraint for settlement or the illiquidity of collateral. This paper adds to this literature by analyzing the distinction between payment instruments in the large value payment system and liquid collateral due to an OTC interbank market.

The distinction between payment instruments and liquid collateral is related to the legal restriction theory of money. Wallace (1983) discusses why money is necessary as the medium of exchange despite the presence of interest-baring Treasury securities. For the reasons, he points out the non-negotiability and the large denomination of these securities in the context of retail payment. This paper brings this question to interbank payment, and shows that the distinction between bank reserves and Treasury securities can be explained by the fact that the interbank market is an OTC market.

Freeman (1996) presents a setup in which debt is repaid with money, and shows that the central bank's discount window improves the efficiency of the resource allocation by allowing banks to swap illiquid IOUs for fiat money. On this issue, Fujiki (2003, 2006) analyzes the effect of liquidity provision policies on cross-border settlement, Mills (2004) proposes an alternative mechanism to the discount window based on collateralized lending, Gu, Guzman and Haslag (2011) analyze the optimal intraday interest rate, and Chapman and Martin (2013) investigate the role of tiering to limit the central bank's exposure to credit risk. This paper complements this literature by analyzing why banks need to swap liquid securities for bank reserves.

Green (1997) discusses whether a private clearing house can take over the role of the central bank in Freeman's model. He argues that there is no intrinsic reason to assume that the constraint facing the central bank is milder than that facing a private clearing house. In this regard, this paper shows that a private clearing house can take over the role of the central bank in the large value payment system, if it has the same commitment ability as the central bank. This paper also discusses the role of the central bank in a private large value payment system when a private clearing house cannot commit to time-inconsistent behavior.

Afonso and Lagos (2014) present a dynamic search model of the OTC federal funds market. They show that the cost of violating a reserve requirement, e.g., a non-negative balance requirement at the end of the day, affects the dynamics of the market because it determines the threat point for bilateral bargaining between banks. This paper derives a similar hold-up problem in decentralized interbank settlement due to a penalty for failed settlement of bank transfers. Given this result, this paper shows that the large value payment system can be interpreted as an implicit interbank settlement contract to prevent a hold-up problem.

This paper's approach to model the large value payment system as an implicit contract is related to the mechanism design approach by Koeppl, Monnet and Temzelides (2008) and Fujiki, Green and Yamazaki (2008). Koeppl, Monnet and Temzelides show that a payment system can implement the optimal resource allocation if agents can rebalance settlement balances at sufficiently high frequency after bilateral exchanges. Fujiki, Green and Yamazaki analyze the optimal design of a payment system given asymmetric information among system participants regarding the probability of settlement failures.

This paper is also related to money-search models with collateral. Shi (1996) shows useless assets except for the owner can serve as collateral to facilitate intertemporal exchange in a money-search model. Berentsen, Camera and Waller (2007) show that a bank can reduce aggregate need for collateral by collecting idle balances and lending them to the demanders of payment instruments. Ferraris and Watanabe (2008) analyze the co-existence of money and credit by introducing loans of money collateralized with illiquid capital. Compared to these papers, this paper's contribution to the literature is to analyze banks sending payments on behalf of their depositors. This feature of interbank payment is captured by the assumption that banks must pay a penalty if they fail to send bank transfers requested by their depositors.

The remainder of the paper is organized as follows. The stylized features of the clearing and settlement system are reviewed in section 2. A model of decentralized interbank settlement is presented in section 3. An interbank settlement contract is introduced into the model in section 4. Section 5 discusses whether a private clearing house can take over the role of the central bank in interbank payment.

2 Stylized features of the clearing and settlement system

The clearing and settlement system processes bank transfers between depositors at different banks in each country. There are usually two tiers in the system. First, small-valued bank transfers from depositors go through an automated clearing house for retail payment. At this tier of the system, the clearing house processes a large number of small-valued bank transfers to calculate the net balance of bank transfers for each bank. Then, at the second tier of the system, a bank originating a net bank transfer settles its position by transferring the same nominal balance to the clearing house's current account at the central bank. This balance is so-called bank reserves. The clearing house passes on the received bank reserves to banks receiving net bank transfers, so that its net position of bank reserves remains zero. This transfer of bank reserves through the clearing house at the second tier clears bank transfers bundled at the first tier.[4]

The central-bank system at the second tier is called a large value payment system, as a balance transfer in the system tends to be large-valued. In fact, if a depositor sends a large-valued bank transfer, then it is directly settled at the second tier without going through the first tier. Examples of the large

[4]Thus, a clearing house acts as a central counterparty.

value payment system operated by the central bank are Fedwire in the U.S., TARGET2 in the Eurozone, CHAPS in the U.K. and BoJ-NET in Japan.

Given this structure of the clearing and settlement system, this paper analyzes whether the central bank is essential for interbank payment at the second tier. For this analysis, this paper presents a parsimonious model to compare the large value payment system operated by the central bank with a decentralized interbank payment system without involvement of the central bank. In this alternative system, banks use safe liquid securities other than bank reserves, such as Treasury securities, as the payment instrument to settle bank transfers. Given no involvement of the central bank, banks negotiate the terms of settlement in an OTC interbank market.

Considering OTC interbank settlement is motivated by the fact that the interbank money market is an OTC market. In reality, each bank has an imbalance between incoming and outgoing transfers of bank reserves each day. To clear these imbalances, banks reallocate bank reserves among them through borrowing and lending in the interbank money market. The central bank does not provide a platform for this market. As a result, banks form an OTC interbank money market by themselves. Thus, the decentralized interbank payment system in the model corresponds to a hypothetical case in which the interbank money market completely replaces the central bank's large value payment system at the second tier of the clearing and settlement system.

The model will show that the liquidity of bank reserves does not explain the need for the central-bank's large value payment system, because there exist other safe liquid securities, such as Treasury securities. Instead, the key role of the central bank is to allow banks to settle outgoing bank transfers unilaterally by sending the same nominal value of bank reserves. This feature of the large value payment system improves efficiency compared to the decentralized interbank payment system.

3 Baseline model of a decentralized interbank payment system

Time is discrete and indexed by $t = 0, 1, 2$. There are two banks indexed by $i = A, B$. Each bank receives a unit amount of goods from its depositors in period 0. For simplicity, the deposit interest rate is set to zero.[5]

Banks can transform deposited goods into loans and bonds. Loans generate an amount R_L (> 1) of goods in period 2 per invested good. Similarly, the gross rate of return on bonds in period 2 is R_B (> 1). Assume that

$$R_L > R_B, \tag{1}$$

so that the rate of return on loans dominates that on bonds. Depositors cannot withdraw goods from banks in period 1, as banks cannot produce any good by terminating loans or bonds in period 1. Thus, the maturity of deposits comes in period 2.

In period 1, each bank i for $i = A, B$ has orders from depositors to send a fraction λ_i of its total deposits to the other bank. The joint probability distribution of λ_A and λ_B is

$$(\lambda_A,\ \lambda_B) = \begin{cases} (\eta,\ 0) & \text{with probability } 0.5, \\ (0,\ \eta) & \text{with probability } 0.5, \end{cases} \tag{2}$$

where $\eta \in (0, 1)$.[6] Note that the two banks are symmetric before the realization of $(\lambda_A,\ \lambda_B)$ in period 1.

If the bank originating bank transfers, i.e., the bank with $\lambda_i = \eta$, fails to settle the bank transfers, then it must incur a cost $\gamma\eta$ ($\gamma > 0$). This cost can be interpreted as representing a long-term cost due to loss of reputation, or a cost payable in period 2 due to a litigation filed by depositors for failed payments. In contrast, the cost of failed settlement of bank transfers for the receiving bank, i.e., the bank with $\lambda_i = 0$, is normalized to zero. Thus, the originating bank must pay a higher penalty for failed settlement of bank transfers than the receiving bank. The underlying assumption is that a deposit contract includes the right to send a bank transfer on demand, for which the originating bank is liable, but the receiving bank is not.

Given this environment, assume that banks cannot commit to any future behavior between them.[7] This assumption implies that banks cannot write

[5] A zero deposit interest rate can be derived as an endogenous equilibrium outcome. See Appendix A for the formal assumption about depositors and a sufficient condition for a zero deposit interest rate and no bank transfer fee in equilibrium.

[6] For simplicity, assume that overlapping gross flows of bank transfers between banks are automatically canceled out, so that banks only need to settle a net flow of bank transfers at the end of period 1.

[7] This assumption can be compatible with each bank's ability to commit to deposit

a pledgeable contract in period 0 to set the terms of settlement of bank transfers in period 1. Even if banks swap some amounts of loans and bonds as collateral between them in period 0, they take an equal amount of collateral from each other, given the ex-ante symmetry between them in period 0. As a result, a bank does not lose anything by reneging on a contract in period 1, because it can cancel out the collateral taken by, and from, the other bank.

Thus, banks need to pay loans or bonds to settle bank transfers between them after the realization of λ_A and λ_B in period 1. Assume that the interbank market is an OTC market; so banks determine the terms of settlement through bilateral bargaining. The outcome of bargaining is determined by Nash bargaining in which each bank has equal bargaining power. If banks do not reach an agreement, then no bank transfer is made. In this case, the originating bank receives a penalty, as assumed above.

Bonds are transferable at no cost between banks. In contrast, if a bank sells its loans to the other bank in period 1, then the bank buying the loans must monitor the loans by itself to generate returns. In this case, the net return per loan in period 2 becomes δ ($\in (0, R_L]$). The difference between R_L and δ is due to a loan monitoring cost.[8] Also, assume that a bank cannot commit to monitoring loans if its loans are submitted to the other bank as collateral for a repo. Thus, a repo and a spot sale are indifferent in the model.

In period 2, each bank receives returns on its loans and bonds, repays deposits given a zero deposit interest rate, and consumes the residual as its profit. Each bank is risk-neutral, and chooses its portfolio of loans and bonds in period 0 to maximize the expected profit in period 2. An equilibrium is a Perfect Bayesian Nash equilibrium for the two banks. See Table 1 for the summary of events in the model.

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contracts. Suppose that depositors can seize loans and bonds in period 2, if a bank defaults on deposit contracts. Denote by v the rate of return on loans and bonds for depositors in case of seizure. Assume $R_L > R_B > 1 + v$. Under this assumption, banks can commit to repaying a deposit interest rate, r_D, up to v. If $v = 0$, then banks can commit to deposit contracts considered in the model.

[8] The loan interest rate, R_L, can be interpreted as the rate of return on loans net of the loan monitoring cost for the originator bank. Thus, this assumption does not imply that an originator bank does not have to monitor loans.

Table 1: Summary of events in the baseline model

Period 0	Period 1	Period 2
There are two banks; each bank receives a unit amount of goods from depositors, given a zero deposit interest rate.	One of the banks has an outflow of bank transfers, η, to the other bank. The probability to be the originating bank is 0.5 for each bank.	Banks receive returns on loans and bonds, repay deposits, and consume the residual.
Banks invest deposited goods into loans and bonds.	A bank must incur a penalty, $\gamma\eta$, if it fails to send bank transfers requested by its depositors within period 1.	The return of goods per loan equals R_L if loans are not transferred in period 1, and δ ($\leq R_L$) if loans are transferred in the period.
	Banks bargain over how much amounts of loans and bonds the originating bank must pay to the receiving bank to settle bank transfers.	The return of goods per bond always equals R_B ($< R_L$).

Hereafter, assume that

Assumption 1. $R_B > \eta (1 + \gamma)$.

Under this assumption, the value of bank transfers, η, is small enough that a bank can always choose to settle bank transfers by investing into a sufficient amount of bonds. Also, assume that the penalty per failed bank transfer, γ, is sufficiently high:

Assumption 2. $\gamma > 4 \left(\dfrac{R_L}{R_B} - 1 \right)$.

This assumption ensures that each bank does not ignore bank-transfer requests from its depositors in any case considered below.

3.1 Efficiency of a decentralized interbank payment system in case of liquid bank loans

For the benchmark, let us start from the case in which loans are transferable at no cost between banks:

Assumption 3. $\delta = R_L$.

Consider the settlement of bank transfers in period 1. Throughout the paper, call the bank with $\lambda_i = \eta$ the "originating bank", and the bank with $\lambda_i = 0$ the "receiving bank". Under Assumption 3, the bargaining problem between the originating and the receiving bank in period 1 takes the following form:

$$\max_{l\in[0,k],\ b\in[0,a]} [-(R_L l + R_B b - \eta) - (-\gamma\eta)]^{0.5}(R_L l + R_B b - \eta)^{0.5}, \tag{3}$$

where: k and a are the amounts of loans and bonds, respectively, held by the originating bank at the beginning of period 1; l and b denote the amounts of loans and bonds, respectively, that the originating bank pays to the receiving bank; and η is the value of bank transfers in the period. In (3), the left square bracket contains the trade surplus for the originating bank, and the right parenthesis contains the trade surplus for the receiving bank. The first term in the left square bracket is a change in profit in period 2 for the originating bank. The second term in the bracket, $-\gamma\eta$, is the penalty for a failed settlement of bank transfers. This penalty determines the threat point for the originating bank.

The solution for the bargaining problem is

$$R_L l + R_B b = \eta + \frac{\gamma\eta}{2}, \tag{4}$$

which is feasible under Assumption 1.[9] This equation implies that the originating bank must pay an extra value of assets, $\gamma\eta/2$, above the value of bank transfers, η. This result is due to bilateral bargaining in the OTC interbank market. The originating bank must complete the bank transfers within period 1 to avoid incurring a penalty, $\gamma\eta$, for failed settlement of bank transfers.

[9] Given Assumption 1 and the flow of funds constraint for each bank in period 0, $k + a = 1$, there exists a pair of l and b satisfying (4), $l \leq k$, and $b \leq a$ for every possible pair of k and a.

The receiving bank takes advantage of this time constraint, charging an extra amount of assets for the settlement of bank transfers.

Now move back to period 0. The profit maximization problem for each bank in the period is:

$$\begin{aligned} \max_{\{k \geq 0,\ a \geq 0\}} \ & R_L k + R_B a - 1 + \frac{1}{2}\frac{\gamma\eta}{2} + \frac{1}{2}\left(-\frac{\gamma\eta}{2}\right), \\ \text{s.t. } & k + a = 1, \end{aligned} \tag{5}$$

where the constraint is a flow of funds constraint that the sum of investments into loans, k, and bonds, a, by each bank in period 0 must equal the total deposits at each bank in the period. The first two terms in the objective function are the returns on loans and bonds in period 2. The third term is the face value of deposit liabilities issued in period 0. The last two terms are the expected net gain and loss due to incoming and outgoing bank transfers, i.e., $\pm(R_L l + R_B b - \eta)$, as implied by (4).

Given $R_L > R_B$, the solution for this problem is

$$(k,\ a) = (1,\ 0). \tag{6}$$

Thus, each bank invests only into the assets with the highest rate of return:

Proposition 1. Suppose Assumption 1 holds. Under Assumption 3, each bank chooses the efficient resource allocation, (6), in period 0.

3.2 Inefficiency of a decentralize interbank payment system in case of illiquid bank loans

The efficiency result described above is overturned if bank loans are illiquid. Now suppose that the cost of liquidating loans, $R_L - \delta$, is sufficiently high:

Assumption 4. $\frac{1}{2}(R_L - \delta)\frac{R_B}{R_L} > (R_L - R_B)$ and $R_L > \delta(1 + \gamma)$.

For a general value of δ, the bargaining problem for the settlement of bank transfers in period 1 takes the following form:

$$\max_{l \in [0,k],\ b \in [0,a]} [-(R_L l + R_B b - \eta) - (-\gamma\eta)]^{0.5}(\delta l + R_B b - \eta)^{0.5}. \tag{7}$$

The left square bracket contains the trade surplus for the originating bank, and the right parenthesis contains the trade surplus for the receiving bank.

Note that the gross rate of return on transferred loans, l, in the right parenthesis is changed from R_L to δ.

Denote by $\theta(a)$ and $\phi(a)$ the net changes in profit for the originating bank and the receiving bank, respectively, as a result of the bargaining problem, (7). Both $\theta(a)$ and $\phi(a)$ are functions of a, given $k = 1 - a$ as implied by the flow of fund constraint for each bank in period 0. Under Assumption 4, the bargaining problem implies

$$(\theta(a),\ \phi(a)) \\ = \begin{cases} (-\gamma\eta,\ 0), & \text{if } R_B a - \eta < -\frac{\delta\gamma\eta}{R_L - \delta}, \\ (-[R_L l(a) + R_B b(a) - \eta],\ \delta l(a) + R_B b(a) - \eta), & \text{otherwise,} \end{cases} \tag{8}$$

where $l(a)$ and $b(a)$ denote the optimal values of l and b, given a:

$$(l(a),\ b(a)) = \begin{cases} \left(\frac{\delta\gamma\eta - (R_L + \delta)(R_B a - \eta)}{2R_L\delta},\ a\right), & \text{if } R_B a - \eta \in \left[-\frac{\delta\gamma\eta}{R_L - \delta},\ \frac{\delta\gamma\eta}{R_L + \delta}\right], \\ (0,\ a), & \text{if } R_B a - \eta \in \left[\frac{\delta\gamma\eta}{R_L + \delta},\ \frac{\gamma\eta}{2}\right], \\ \left(0,\ \frac{1}{R_B}\left(\eta + \frac{\gamma\eta}{2}\right)\right), & \text{if } R_B a - \eta > \frac{\gamma\eta}{2}. \end{cases} \tag{9}$$

See Appendix B for the proof for (8) and (9).

These equations imply that banks fail to agree on the settlement of bank transfers (i.e., $\theta(a) = -\gamma\eta$), if a is too small. This result holds because the cost of liquidating loans, $R_L - \delta$, is too large under Assumption 4. If a is sufficiently large for banks to settle bank transfers, then the value of loan transfer, l, is weakly decreasing in a. In this case, the originating bank must pay a higher value of assets than the value of bank transfers, i.e., $R_L l(a) + R_B b(a) > \eta$, whether $l(a)$ is positive or zero. This result is due to bilateral bargaining in the OTC interbank market: the receiving bank takes advantage of the constraint that the originating bank must complete the bank transfers within period 1 to avoid a penalty. This hold-up problem is as same as the reason behind the second term on the right-hand side of (4).

Given (8) and (9), the profit maximization problem for each bank in period 0 can be written as

$$\begin{aligned} \max_{\{k \geq 0,\ a \geq 0\}} \ & R_L k + R_B a - 1 + \frac{1}{2}\theta(a) + \frac{1}{2}\phi(a'), \\ \text{s.t. } & k + a = 1, \end{aligned} \tag{10}$$

where a' denotes the amount of bonds held by the other bank at the end of the period, which is taken as given.

Under Assumptions 2 and 4, banks invest into a just enough amount of bonds in period 0 to avoid liquidation of loans in period 1:

Proposition 2. Suppose Assumptions 1 and 2 hold. Under Assumption 4, each bank chooses

$$(k,\ a) = \left(1 - a,\ \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L + \delta}\right)\right), \tag{11}$$

in period 0. Given this value of a, the originating bank pays no loan to the receiving bank for the settlement of bank transfers in period 1:

$$(l,\ b) = \left(0,\ \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L + \delta}\right)\right). \tag{12}$$

Proof. See Appendix C. □

As implied by (8), each bank can reduce the amount of bonds necessary to settle bank transfers by limiting its bond holdings, a, ex-ante, while maintaining $l(a) = 0$. The liquidity-saving effect of limiting the ex-ante bond holdings is not perfect, however, as the originating bank still has to pay an extra value of bonds above the value of bank transfers, η, as implied by (12). This effect of bilateral bargaining in an OTC interbank market increases the amount of bonds that each bank must invest into in period 0.

4 Public interbank payment system operated by the central bank

Now introduce the central bank into the baseline model. Two cases will be considered. In the first case, the central bank issues bank reserves just as liquid assets. In the second case, the central bank is introduced as the custodian of collateral in an interbank settlement contract. It will be shown that the central bank can improve the efficiency of the payment system only in the second case.

4.1 No efficiency gain from the introduction of bank reserves just as liquid store of wealth

Suppose that the central bank allows each bank to exchange its bonds for bank reserves in period 0. The central bank repays bank reserves by the whole return on the bonds in period 2. Thus, the central bank just holds bonds on behalf of banks. Bank reserves are transferable between banks at no cost in period 1, just like bonds. The central bank cannot accept a transfer of loans from banks, because it does not have enough ability to monitor loans.

In this case, bank reserves and bonds are identical as liquid store of wealth. Thus, the bargaining problem over the settlement of bank transfers in period 1 remains essentially the same as (3) and (7) under Assumptions 3 and 4, respectively.[10] Hence, Propositions 1 and 2 remain to hold.

4.2 Introduction of the central bank as the custodian of collateral in an interbank settlement contract

Next, suppose that the central bank offers an interbank settlement contract in period 0. A contract,

$$f : (\hat{\lambda}_A,\ \hat{\lambda}_B) \in \{\emptyset,\ \lambda_A\} \times \{\emptyset,\ \lambda_B\} \mapsto \{b_A(\hat{\lambda}_A,\ \hat{\lambda}_B),\ b_B(\hat{\lambda}_A,\ \hat{\lambda}_B)\} \in \mathbb{R}^2_+, \tag{13}$$

maps the outflows of bank transfers reported by bank A, $\hat{\lambda}_A$, and bank B, $\hat{\lambda}_B$, to a contingent flow of bonds, $b_i(\hat{\lambda}_A,\ \hat{\lambda}_B)$, from bank i to the central bank for $i = A, B$. A negative value of $b_i(\hat{\lambda}_A,\ \hat{\lambda}_B)$ indicates a flow of bonds from the central bank to bank i. If $\hat{\lambda}_i = \emptyset$, then it implies that bank i opts out of the contract in period 1. Otherwise, $\hat{\lambda}_i = \lambda_i$, that is, bank i reports bank-transfer requests from its depositors truthfully. The central bank does

[10] Only the following changes in the notations are necessary: $R_B a$ is redefined as the sum of the par value of bonds and bank reserves held by each bank in period 1; and b is redefined as the sum of the par value of bonds and bank reserves transferred in period 1.

not have any endowment in any period; thus:

$$\sum_{i=A,B} b_i(\hat{\lambda}_A,\ \hat{\lambda}_B) = 0 \quad \text{for all } (\hat{\lambda}_A,\ \hat{\lambda}_B). \tag{14}$$

The equality implies that the net flow of bonds for the central bank must be always zero.

The central bank offers only a symmetric contract between banks, given their ex-ante symmetry in period 0. Hence:

$$\hat{b} \equiv b_A(\eta,\ 0) = b_B(0,\ \eta), \tag{15}$$

$$b_A(0,\ \eta) = b_B(\eta,\ 0) = -\hat{b}, \tag{16}$$

where $\hat{b}$ denotes the value of bonds to be transfered from the originating bank to the receiving bank. The central bank aims to maximize each bank's expected profit in period 2.

To implement a contract, the central bank requires each bank to submit an amount $\hat{b}$ of bonds in period 0. Then the central bank transfers bond balances between banks according to the contract in period 1. Assume that the central bank can commit to returning the resulting balance of bonds to each bank only in period 2, even if a bank opts out of the contract in period 1. It cannot accept loans held by banks as collateral, as it does not have enough ability to monitor loans.

To maintain consistency with the baseline model, assume that banks cannot commit to any future behavior:

Assumption 5. If either bank rejects the offer of a contract in period 0, or opts out of a contract in period 1, then banks settle bank transfers through bilateral bargaining in period 1.

Thus, the central bank cannot enforce a contract if either bank has a higher ex-post profit in bilateral bargaining in period 1 than under the contract. See Table 2 for the summary of the model with an interbank settlement contract offered by the central bank.

4.3 Optimal interbank settlement contract

Under Assumption 5, a contract must ensure that the receiving bank does not incur a loss from receiving a bank transfer, because the bank would not

Table 2: Summary of the model with an interbank settlement contract

Period 0	Period 1	Period 2
There are two banks; each bank receives a unit amount of goods from depositors, given a zero deposit interest rate.	One of the banks has an outflow of bank transfers, η, to the other bank. The probability to be the originating bank is 0.5 for each bank.	If banks enter into a contract in period 0, the central bank returns the remaining balance of bonds to each bank.
Banks invest deposited goods into loans and bonds.	A bank must incur a penalty, $\gamma\eta$, if it fails to send bank transfers from its depositors within period 1.	Banks receive returns on loans and bonds, repay deposits, and consume the residual.
The central bank offers an interbank settlement contract for banks, which requires each bank to submit bonds to the central bank in period 0.	If neither bank rejected the offer of a contract in period 0 or opts out of a contract in period 1, then the central bank transfers bond balances between banks according to bank-transfer requests reported by each bank, as specified by the contract.	The return of goods per loan equals R_L if loans are not transferred in period 1, and δ ($< R_L$) if loans are transferred in the period.
	Otherwise, banks bargain over how much amounts of loans and bonds the originating bank must pay to the receiving bank to settle bank transfers.	The return of goods per bond always equals R_B ($< R_L$).

incur a loss even if no bank transfer were settled. Thus,

$$R_B\hat{b} - \eta \geq 0, \text{ for } i = A, B, \tag{17}$$

where the left-hand side is the net gain in profit for the receiving bank in case that it stays in the contract in period 1.

For now, suppose that the originating bank does not have incentive to opt out of the contract in period 1. This conjecture will be verified later. Given this conjecture, the optimal contract problem for the central bank is specified as follows:

$$\begin{aligned}
\max_{\{k\geq 0,\, a\geq 0,\, \hat{b}\}} \; & R_L k + R_B a - 1 - \frac{1}{2}(R_B\hat{b} - \eta) + \frac{1}{2}(R_B\hat{b} - \eta), \\
\text{s.t. } & k + a = 1, \\
& R_B\hat{b} - \eta \geq 0, \\
& a \geq \hat{b},
\end{aligned} \tag{18}$$

where: k and a are the amounts of loans and bonds, respectively, that each bank invests into in period 0; and $\hat{b}$ is the amount of bonds to be transferred from the originating bank to the receiving bank in period 1. The first constraint is the flow of funds constraint for each bank in period 0. The second constraint is the incentive-compatibility constraint for the receiving bank to remain in the contract, (17). The third constraint indicates that each bank must invest into the amount of bonds to submit to the central bank, $\hat{b}$, in period 0 under the contract. It is straightforward to show that the solution to this problem is characterized by

$$(k,\ a,\ \hat{b}) = \left(1 - a,\ \hat{b},\ \frac{\eta}{R_B}\right), \tag{19}$$

which is feasible under Assumption 1.

Now confirm that the bank with $\lambda_i = \eta$ does not have incentive to deviate from this contract in period 1. Suppose that banks enter into the contract characterized by (19) in period 0, but one of the banks opts out of the contract to initiate bilateral bargaining in period 1. In this case, banks can transfer only loans between them, because the central bank keeps their entire bond holdings until period 2 given $a = \hat{b}$. Thus, the bargaining problem in this case is

$$\max_{\tilde{l}\in[0,k]} [-(R_L\tilde{l} - \eta) - (-\gamma\eta)]^{0.5}(\delta\tilde{l} - \eta)^{0.5}, \tag{20}$$

where: the left square bracket and the right parenthesis are the trade surpluses for the originating bank and the receiving bank, respectively; and $\tilde{l}$ is the amount of loans transfered from the originating bank to the receiving bank.

Under Assumption 4, banks do not have gains from trade in bilateral bargaining, because the cost of liquidating loans, $R_L - \delta$, is too high and banks hold no bond to transfer in period 1. Also, note that the contract characterized by (19) leaves each bank break-even whether originating or receiving bank transfers, because $R_B \hat{b} = \eta$. Thus, no bank is strictly better off by opting out of the contract and not settling bank transfers. Hence, no bank has incentive to opt out of the contract in period 1.

In summary, given Assumptions 2 and 4, banks participate into the optimal contract, (19), in period 0 because they can save the amount of bonds necessary for the settlement of bank transfers, as implied by (11). Hence:

Proposition 3. Suppose that Assumptions 1, 2, 4, and 5 hold. Banks participate into the interbank settlement contract characterized by (19).

Note that the central bank does not need to commit to confiscating the bonds of a bank opting out of the contract. To implement the contract, it only needs to retain bonds until the end of period 1. Thus, $b_i(\emptyset, \cdot) = b_i(\cdot, \emptyset) = 0$ for $i = A, B$ in the optimal contract.

4.4 Interpretation of the large value payment system as an implicit interbank settlement contract

In the large value payment system operated by the central bank, a bank can settle outgoing bank transfers unilaterally by sending the same nominal value of bank reserves to the receiving banks. This feature of the large value payment system is shared by the interbank settlement contract in the model, as the contract allows banks to settle bank transfers without bargaining. Also, banks in reality obtain bank reserves in exchange for liquid assets, such as Treasury securities, through open market operations. In light of the model, this transaction can be interpreted as the submission of liquid collateral to the central bank under an interbank settlement contract. This result explains why banks swap Treasury securities for bank reserves despite that both are the liquid liabilities of the consolidated government.

In addition, the model can explain the fact that banks are not bound to stay in the large value payment system by any explicit contract. In the model, the central bank can implement the optimal interbank settlement contract as an implicit contract only by retaining collateral submitted by banks until the end of the settlement of bank transfers. This result is consistent with the fact that the large value payment system typically does not include a rule to confiscate bank reserves in case of a settlement failure.

4.5 Liquidity-saving effect of the floor system

In the model, each bank submits to the central bank an enough amount of bonds to settle any possible bank transfer by the contract. Thus, banks do not settle any bank transfer in an OTC interbank market. While this result may seem contradictory to the existence of the interbank money market in reality, in which banks settle an imbalance of bank transfers through bilateral bargaining, the optimal contract in the model corresponds to the floor system. This system is a type of monetary policy. In the floor system, the central bank supplies a sufficient amount of bank reserves for interbank payment in advance, so that banks do not need to borrow from the interbank money market. To give banks incentive to hold the supplied bank reserves, the central bank pay interest on bank reserves. Consequently, this interest rate determines the short-term nominal interest rate in the financial market.[11]

Note that the central bank in the model passes to banks the whole return on bonds received as collateral. This policy is equivalent to interest payment on bank reserves. Moreover, the interest paid by the central bank equals that on bonds, which can be interpreted as a market interest rate. Thus, the model illustrates that the floor system has a liquidity-saving effect through eliminating the need for the OTC interbank money market.

5 Discussion and conclusions: Can a private clearing house provide a large value payment system on behalf of the central bank?

A remaining question is whether the custodian of collateral in an interbank settlement contract must be the central bank, or can be a private clearing house. For this question, the key characteristic of the central bank in the model is its ability to commit to returning the remaining balance of bonds

[11]The floor system has been adopted by New Zealand since July 2006.

to each bank after the settlement of bank transfers. Note that this is a commitment to an time-inconsistent policy, as the central bank must return bonds to banks in exchange for nothing ex-post. If the custodian of collateral cannot commit to returning collateral, then the interbank settlement contract breaks down.

This result is consistent with the fact that the central bank absorbs bank reserves through open market operations, despite that bank reserves are worthless for the central bank. If a private clearing house can show the same behavior in any circumstance, then it can take over the role of the central bank in the large value payment system.

A failure to return collateral, however, is a real concern. For example, it is called a settlement fail in the repo market, and is a regular incident in the market.[12] In particular, there was a large number of settlement fails for U.S. Treasury securities in 2008 when the holders of safe collateral were not willing to return the collateral amid the collapse of Bear Stearns and Lehman Brothers. This experience suggests that even though a private entity may be able to commit to returning collateral in normal time by reputation concerns, its commitment can be ineffective at the time of financial stress.

If this is the case, a private clearing house needs some commitment device to operate a large value payment system on behalf of the central bank. In this regard, bank reserves are ideal collateral for a private clearing house, as they cannot be taken out of the central bank's large value payment system. Thus, the role of the central bank as the custodian of collateral would remain in this case. This implication of the model is consistent with the current observation that the CHIPS and the CLS use bank reserves as collateral.

It remains an open question whether how much fractions of interbank payments will take place in a private large value system. One advantage of a private system is the fast adoption of new technology to save the operational cost. On the other hand, if banks' collateral is ultimately held by the central bank, then adding a private clearing house as another layer between the central bank and banks may be inefficient. Analyzing this trade-off is left for future research.

[12] See Garbade and Fleming (2005) for more details.

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A Sufficient condition for a zero deposit interest rate and no bank transfer fee

A.1 Formal assumption about depositors

First of all, the following environment for depositors is implicitly assumed throughout this paper.

Each of the two banks has a unit continuum of depositors. Each depositor is endowed with a unit of goods in period 0. In each period, depositors can store goods by themselves until the next period without depreciation. Assume that

$$R_L > R_B > 1, \tag{21}$$

where one equals the gross rate of return on storage. Alternatively, depositors can deposit goods at each depositor's bank in period 0. Depositors cannot withdraw goods from banks in period 1, as banks cannot produce any good by terminating loans or bonds in period 1. Thus, the maturity of deposits comes in period 2.

Each depositor becomes a buyer or a seller due to an idiosyncratic shock in period 1. A seller can produce goods at a unit marginal utility cost per good in period 1, and consume goods in period 2. A buyer can consume goods produced by sellers at the other bank in period 1. A buyer cannot consume goods in period 2. Each depositor maximizes the expected utility of consumption:

$$U = p_1 \rho c_{b,1} + (1 - p_1)(-h_{s,1} + c_{s,2}), \tag{22}$$

where: ρ (> 0) is a weight on the utility of consumption in case of becoming a buyer; p_1 is the probability to be a buyer in period 1 for each depositor in period 0; $c_{b,1}$ is the consumption in period 1 in case of becoming a buyer; and $h_{s,1}$ and $c_{s,2}$ are the production in period 1 and the consumption in period 2, respectively, in case of becoming a seller.

The goods market in period 1 is competitive; thus the relative price of goods in period 1 to goods in period 2 equals one, given a unit marginal utility cost of production as implied by the utility function, (22). To pay the price for goods in period 1, buyers can order banks to send the equivalent par value of deposits from their accounts to sellers' accounts in period 1.

Thus, the value of ρ represents the shadow value of liquidity when depositors become buyers.

The buyer fraction of depositors at each bank is stochastic. At each bank, a fraction λ_i of depositors become buyers, and order the bank to transfer their whole deposits in period 1. The joint probability distribution of λ_A and λ_B is

$$(\lambda_A,\ \lambda_B) = \begin{cases} (\eta,\ 0) & \text{with probability } 0.5, \\ (0,\ \eta) & \text{with probability } 0.5, \end{cases} \tag{23}$$

where $\eta \in (0, 1)$. Given (2), the unconditional probability for each depositor to be a buyer, i.e., p_1, is

$$p_1 = 0.5\eta. \tag{24}$$

If banks fail to agree on the settlement of bank transfers, then no bank transfer is made, and the transactions between buyers and sellers in period 1 are canceled. In this case, the bank with an outflow of bank transfers, i.e., $\lambda_i = \eta$, must incur a cost γ (> 0) per depositor requesting a bank transfer. The bank receiving bank transfers does not have to incur such a cost, because it does not have any contract with depositors at the other bank.

Each bank sets the deposit interest rate and the bank transfer fee for its depositors monopolistically in period 0, given an assumption that depositors cannot change their banks. Banks cannot charge different bank transfer fees for buyers and sellers, because each depositor's type is private information for the depositor. All depositors choose deposits in period 0 if they are indifferent between deposits and storage.

A.2 Sufficient condition for a zero deposit interest rate and no bank transfer fee

Given this environment for depositors, assume that the shadow value of liquidity for buyers is sufficiently high:

Assumption 6. ρ is arbitrarily large.

Under this assumption, the following proposition holds:

Proposition 4. Denote the deposit interest rate between periods 0 and 2 by r_D and the bank transfer fee by f. Under Assumption 6,

$$r_D = f = 0, \tag{25}$$

in any equilibrium in the baseline model and the model with an interbank settlement contract.

Proof.

Participation constraint for depositors. Suppose that a bank sets a negative bank transfer fee in period 1. In this case, buyers can earn an excess return over storage. Sellers can also earn an excess return, because banks offer the same bank transfer fee for all depositors and sellers can buy goods with bank transfers in period 1 and store the goods until period 2. The presence of an excess return for all depositors implies that banks are not maximizing its profit. Thus, the bank transfer fee must be non-negative:

$$f \geq 0, \tag{26}$$

where f denotes the bank transfer fee.

The form of a depositor's utility maximization problem depends on whether its bank will settle its bank transfers. First, suppose that banks settle their depositors' bank transfers in period 1 in equilibrium. Given the deposit interest rate, r_D, and the bank transfer fee satisfying (26), a depositor's utility maximization problem in this case can be written as

$$\begin{aligned}
\max_{\{d_0 \geq 0,\, x_0 \geq 0,\, h_{s,1} \geq 0\}} \quad & p_1 \rho c_{b,1} + (1 - p_1)(-h_{s,1} + c_{s,2}), && (27)\\
\text{s.t.} \quad & d_0 + x_0 = 1, && (28)\\
& c_{b,1} = (1 + r_D - f) d_0 + x_0, && (29)\\
& c_{s,2} = (1 + r_D) d_0 + x_0 + h_{s,1}, && (30)
\end{aligned}$$

where d_0 and x_0 denote the values of deposits and storage that the depositor invests into in period 0. The first constraint is the flow of funds constraint in period 0. The second and the third constraint are the flow of funds constraints in period 1 in case of becoming a buyer and a seller, respectively. In the second constraint, a buyer can buy an amount of goods equal to the par value of his deposits minus the bank transfer fee, given that the competitive relative price of goods in period 1 to goods in period 2 is one. In the third constraint, $h_{s,1}$ equals the par value of deposits that a seller receives for his products in period 1. As implied by the maximization problem, a seller is indifferent to the value of $h_{s,1}$. Thus, $h_{s,1}$ absorbs the demand for goods in period 1 to clear the goods market at the competitive goods price. A depositor chooses deposits over storage in period 0, if and only if r_D and f satisfies

$$p_1\rho(1 + r_D - f) + (1 - p_1)(1 + r_D) \geq p_1\rho + 1 - p_1. \tag{31}$$

Next, suppose that banks do not settle bank transfers in period 1 in equilibrium. By a similar calculation to the previous case, depositors at such a bank choose deposits over storage if and only if

$$(1 - p_1)(1 + r_D) \geq p_1\rho + 1 - p_1. \tag{32}$$

Bilateral bargaining between banks in period 1. Given the participation into deposit contracts by depositors under (26) and (31), solve the model backward. The bargaining problem between banks for the settlement of bank transfers in period 1 takes the following form:

$$\begin{aligned} \max_{l\in[0,k],\, b\in[0,a]} & \{-[R_L l + R_B b - (1 + r_D - f)\eta] - (-\gamma\eta)\}^{0.5} \\ & \cdot [\delta l + R_B b - (1 + r_D - f)\eta]^{0.5} \end{aligned} \tag{33}$$

where: k and a are the amounts of loans and bonds, respectively, held by the bank with an outflow of bank transfers, i.e., $\lambda_i = \eta$; l and b denote the amounts of loans and bonds, respectively, paid by the bank; and $(1+r_D-f)\eta$ is the flow of deposit liabilities due to bank transfers, given r_D and f set by the bank in period 0. The left curly and the right square bracket contain the trade surpluses for the banks with $\lambda_i = \eta$ and $\lambda_i = 0$, respectively. The first term in the left curly bracket is a change in the net worth of the bank with $\lambda_i = \eta$. The second term in the bracket, $\gamma\eta$, is the cost of a settlement failure for the bank, which determines the bank's threat point.

If $\delta = R_L$, then the first-order conditions with respect to l and b imply

$$R_L l(a) + R_B b(a) = (1 + r_D - f)\eta + \frac{\gamma\eta}{2}, \tag{34}$$

where $l(a)$ and $b(a)$ denote the solutions for l and b, respectively, for a given value of a. Satisfying this equation is feasible under any value of a (i.e., $l(a) \in (0, k)$ and $b(a) \in (0, a)$ given $k + a = 1$), given Assumption 1.

If $\delta < R_L$, the first-order conditions with respect to l and b are:

$$-\frac{R_L}{-[R_L l + R_B b - (1 + r_D - f)\eta] + \gamma\eta} + \frac{\delta}{\delta l + R_B b - (1 + r_D - f)\eta} + \underline{\theta}_l - \overline{\theta}_l = 0, \quad (35)$$

$$-\frac{R_B}{-[R_L l + R_B b - (1 + r_D - f)\eta] + \gamma\eta} + \frac{R_B}{\delta l + R_B b - (1 + r_D - f)\eta} - \overline{\theta}_b = 0, \quad (36)$$

where $\underline{\theta}_l$, $\overline{\theta}_l$, and $\overline{\theta}_b$ are proportional to the non-negative Lagrange multipliers for $0 \leq l$, $l \leq k$, and $b \leq a$. The other constraint, $0 \leq b$, is always slack.

Given that the denominator in each side is the same across the two conditions and the assumption that $R_L \geq \delta$, $\underline{\theta}_l = \overline{\theta}_l = \overline{\theta}_b = 0$ cannot hold. Thus, there are four cases to consider: $\{l = 1 - a,\ b = a\}$; $\{l \in (0,\ 1 - a),\ b = a\}$; $\{l = 0,\ b = a\}$; and $\{l = 0,\ b \in (0, a)\}$.

In the first case, $\underline{\theta}_l = 0$ and $\overline{\theta}_l \geq 0$. For this case to happen,

$$\frac{R_L}{-[R_L(1 - a) + R_B a - (1 + r_D - f)\eta] + \gamma\eta} \leq \frac{\delta}{\delta(1 - a) + R_B a - (1 + r_D - f)\eta}. \quad (37)$$

Given (36) and the assumption $R_L \geq \delta$, $\overline{\theta}_b > 0$.

In the second case, $\underline{\theta}_l = \overline{\theta}_l = 0$. In this case, (35) implies that

$$\exists l \in (0,\ 1 - a),\ \text{s.t.}\ \frac{R_L}{-[R_L l + R_B a - (1 + r_D - f)\eta] + \gamma\eta} = \frac{\delta}{\delta l + R_B a - (1 + r_D - f)\eta}. \quad (38)$$

Given (36) and the assumption $R_L \geq \delta$, $\overline{\theta}_b > 0$.

In the third case, $\underline{\theta}_l \geq 0$, $\overline{\theta}_l = 0$, and $\underline{\theta}_b \geq 0$. Thus, (36) implies

$$\frac{R_B}{-[R_B a - (1 + r_D - f)\eta] + \gamma\eta} \geq \frac{R_B}{R_B a - (1 + r_D - f)\eta}. \quad (39)$$

This condition is sufficient for (35), given the assumption $R_L \geq \delta$.

In the fourth case, $\underline{\theta}_l \geq 0$, $\overline{\theta}_l = 0$, and $\underline{\theta}_b = 0$. Hence:

$$\exists b \in (0,\ a),\ \text{s.t.}\ \frac{R_B}{-[R_B b - (1 + r_D - f)\eta] + \gamma\eta} = \frac{R_B}{R_B b - (1 + r_D - f)\eta}. \tag{40}$$

This condition is sufficient for (35), given the assumption $R_L \geq \delta$.

Summarizing the four cases, the solutions for l and b under $\delta < R_L$ take the following form:

$$(l(a),\ b(a)) = \begin{cases} (1-a,\ a)\,, & \text{if } R_B a - (1 + r_D - f)\eta \\ & \qquad \leq \frac{\delta\gamma\eta - 2R_L\delta(1-a)}{R_L+\delta}, \\ \left(\frac{\delta\gamma\eta - (R_L+\delta)[R_B a - (1+r_D-f)\eta]}{2R_L\delta},\ a\right), & \text{if } R_B a - (1 + r_D - f)\eta \\ & \qquad \in \left(\frac{\delta\gamma\eta - 2R_L\delta(1-a)}{R_L+\delta},\ \frac{\delta\gamma\eta}{R_L+\delta}\right), \\ (0,\ a)\,, & \text{if } R_B a - (1 + r_D - f)\eta \\ & \qquad \in \left[\frac{\delta\gamma\eta}{R_L+\delta},\ \frac{\gamma\eta}{2}\right], \\ \left(0,\ \frac{1}{R_B}\left[(1 + r_D - f)\eta + \frac{\gamma\eta}{2}\right]\right), & \text{if } R_B a - (1 + r_D - f)\eta > \frac{\gamma\eta}{2}, \end{cases} \tag{41}$$

if both banks have non-negative trade surpluses in each case.

In the third and the fourth case, it is immediate that both banks have non-negative trade surpluses. In the second case, the necessary and sufficient condition for non-negative trade surpluses for both banks is

$$\delta\gamma\eta + (R_L - \delta)[R_B a - (1 + r_D - f)\eta] \geq 0. \tag{42}$$

If a falls into the range for the second case and the values of r_D and f violate this condition, then banks choose not to settle bank transfers. In the first case, the necessary and sufficient conditions for non-negative trade surpluses are:

$$\gamma\eta \geq R_L(1-a) + R_B a - (1 + r_D - f)\eta, \tag{43}$$

$$\delta(1-a) + R_B a - (1 + r_D - f)\eta \geq 0. \tag{44}$$

If (42) and (43)-(44) are not satisfied in the second and the first case, respectively, then banks do not settle bank transfers in period 1.

Profit maximization problem for each bank in period 0. Now specify the profit maximization problem for each bank in period 0. As implied by (41)-(44), whether a bank settles an outflow of bank transfers in period 1 depends on the bank's choice of a, r_D, and f in period 0.

First, suppose that a bank chooses such values of a, r_D, and f that it will not settle an outflow of bank transfers in period 1. In this case, the bank's profit maximization becomes

$$\begin{aligned} \max_{\{k\geq 0,\, a\geq 0,\, r_D,\, f\}} \; & R_L k + R_B a - (1+r_D)(k+a) \\ & - \frac{1}{2}\gamma\eta + \frac{1}{2}\Theta(a',\, r_D',\, f') \\ \text{s.t. } & k + a = 1, \\ & (1-p_1)(1+r_D) \geq p_1\rho + 1 - p_1, \end{aligned} \tag{45}$$

where $\gamma\eta$ is the penalty incurred by the bank when the bank fails to settle an outflow of bank transfers, and

$$\Theta(a,\, r_D,\, f) \equiv \begin{cases} \delta l(a) + R_B b(a) - (1 + r_D - f)\eta & \text{if trade surpluses are non-negative,} \\ 0, & \text{otherwise.} \end{cases} \tag{46}$$

Thus, $\Theta(a,\, r_D,\, f)$ is a function returning the profit that a bank transfers to the other bank when it has an outflow of bank transfers. In (45), a', r_D', and f' denote the other bank's bond holdings, deposit interest rate, and bank transfer fee, respectively, for each bank. Thus, each bank takes as given the incoming transfer of assets in case of an inflow of bank transfers, $\Theta(a',\, r_D'\, f')$.

The first constraint is the flow of funds constraint in period 0. The second constraint is (32), the participation constraint for depositors given no settlement of their bank transfers to sellers. Under Assumption 6, r_D must be arbitrarily high to satisfy (32). Thus, a bank's expected profit is arbitrarily close to $-\infty$ if it chooses such values of a, r_D, and f that it will not settle an outflow of bank transfers.

Next, suppose that a bank chooses such values of a, r_D, and f that it will settle an outflow of bank transfers in period 1. In this case, the bank's profit maximization problem becomes

$$
\begin{aligned}
\max_{\{k\geq 0,\ a\geq 0,\ r_D,\ f\}} \quad & R_L k + R_B a - (1+r_D)(k+a) \\
& -\frac{1}{2}\Gamma(a,\ r_D,\ f) + \frac{1}{2}\Theta(a',\ r'_D,\ f') \\
\text{s.t. } & k + a = 1, \\
& p_1\rho(1+r_D-f) + (1-p_1)(1+r_D) \geq p_1\rho + 1 - p_1, \\
& f \geq 0,
\end{aligned} \tag{47}
$$

where $\Gamma(a,\ r_D,\ f)$ is a function for a loss of profit when a bank has an outflow of bank transfers:

$$
\Gamma(a,\ r_D,\ f) \equiv R_L l(a) + R_B b(a) - (1+r_D-f)\eta. \tag{48}
$$

In (47), the first constraint is the flow of funds constraint in period 0; the second constraint is (31), the participation constraint for depositors given that their bank transfers will be settled; and the third constraint is (26), the no-arbitrage condition for depositors.

At the maximum, (31) holds in equality. Thus, Assumption 6 implies that

$$
1 + r_D - f \simeq 1. \tag{49}
$$

Accordingly, the effect of a change in f on l and b is arbitrarily close to 0 for any choice of a, as implied by (34) and (41). Thus, it is always optimal to set (25) to minimize the deposit interest rate, r_D, given (26), whether banks settle bank transfers in period 1 or not. The expected profit from this behavior is bounded below, given that the value of $\Theta(a',\ r'_D\ f')$ is always non-negative as implied by (46).

In case of an interbank settlement contract. A similar result holds in case of an interbank settlement contract described in Section 4. Given r_D and f, the amount of bonds that must be held by each bank in period 0 under the contract, (19), is replaced with

$$
a = \hat{b} = \frac{(1+r_D-f)\eta}{R_B}. \tag{50}
$$

$$
\tag{51}
$$

Thus, a and $\hat{b}$, and hence k, are insensitive to f, given (49). Hence, raising f only increases r_D. □

B Proof for (8) and (9)

The first-order conditions for the bargaining problem between banks in period 1 imply (41) with the conditions for non-negative trade surpluses, (42)-(44), as shown in Appendix A. Given Assumption 6, substitute (25) into (41) and (42)-(44).

Now consider the bargaining problem, (7), without the constraint $l \leq 1-a$. In this case, the solution for l and b falls into the second, the third, or the fourth case of (41) on the condition that both banks have non-negative trade surpluses. In the third and the fourth case, banks always have non-positive trade surpluses. In the second case, (42) must be satisfied. Thus, if l is not constrained by $l \leq 1-a$, then the solution falls into the second case if and only if

$$R_B a - \eta \in \left[-\frac{\delta\gamma\eta}{R_L - \delta}, \frac{\delta\gamma\eta}{R_L + \delta} \right]. \tag{52}$$

If $R_B a - \eta$ is less than the lower bound of this range, then banks cannot have non-negative trade surpluses for both at the same time in the bargaining problem, (7), without the constraint $l \leq 1-a$. Thus, banks make no deal in this case, even if there exists the constraint.

For (8), it remains to show that the constraint $l \leq 1-a$ does not bind if a satisfies (52). In the second case of (41), $l(a) \leq 1-a$ is satisfied if and only if

$$\frac{\delta\eta(1+\gamma) + R_L\eta}{2R_L\delta} \leq \frac{(R_L+\delta)R_B}{2R_L\delta} a + 1 - a. \tag{53}$$

If $(R_L+\delta)R_B \leq 2R_L\delta$, then this condition holds for any value of a satisfying (52), because the right-hand side of the condition is decreasing in a, and $l(a) = 0$ at the upper bound of (52).

The condition (53) also holds if $(R_L+\delta)R_B > 2R_L\delta$, because the lowest value of a satisfying (53) is lower than the lower bound of (52):

$$\begin{aligned} &\frac{1}{R_B}\left(\eta - \frac{\delta\gamma\eta}{R_L - \delta}\right) - \frac{[(1+\gamma)\delta + R_L]\eta - 2R_L\delta}{(R_L+\delta)R_B - 2R_L\delta} \\ \propto\ & -\gamma\eta(R_B - \delta) + (R_L - \delta)(R_B - \eta) \\ >\ & (R_L - \delta)[R_B - \eta(1+\gamma)] \\ >\ & 0, \end{aligned} \tag{54}$$

where: the first line is proportional to the second line given $R_L > \delta(1+\gamma)$ under Assumption 4; the second inequality is due to the assumption that $R_L > R_B$; and the last inequality holds under Assumptions 1 and 4. Overall, the constraint $l \leq 1 - a$ never binds if a satisfies (52). $\square$

C Proof for Proposition 2

If $l(a) = 0$ at the optimum of the bargaining problem (10), then each bank minimizes its bond holdings, a, because an increase in the bond holdings only results in a transfer of more bonds in case of an outflow of bank transfers, as implied by (8).

If the value of a falls into the first case of (8), i.e., $l(a) > 0$, then the expected profit for a bank in period 0 is increasing in a given the first condition in Assumption 4. Thus, if a bank settles an outflow of bank transfers in period 1, then the bank chooses (11) at the optimum in period 0.

If no settlement of an outflow of bank transfers is optimal for each bank, then each bank sets $a = 0$. Given $\phi(a')$, the expected profit in this case is lower than the one under (11), because:

$$R_L - \left(\frac{R_L}{R_B} - 1\right)\left(\eta + \frac{\delta\gamma\eta}{R_L + \delta}\right) - 1 - \frac{1}{2}\frac{\delta\gamma\eta}{R_L + \delta} + \phi(a') > R_L - 1 - \frac{1}{2}\gamma\eta + \phi(a'), \quad (55)$$

where the left-hand and the right-hand side are the expected profits for a bank in period 0 when the bank chooses (11) and $a = 0$, respectively. Note that (55) is equivalent to

$$\frac{\gamma R_L R_B}{2} - (R_L - R_B)\left[R_L + \delta(1+\gamma)\right] > 0, \quad (56)$$

which holds under Assumption 2 and the second condition in Assumption 4. $\square$