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Abstract

This paper compares fiat money and a Lucas' tree in an overlapping generations model. A Lucas' tree with a positive dividend has a unique competitive equilibrium price. Moreover, the price converges to the monetary equilibrium value of fiat money as the dividend goes to zero in the limit. Thus, the value of liquidity represented by a rational bubble is part of the fundamental price of a standard interest-bearing asset. A Lucas' tree has multiple equilibrium prices if the dividend vanishes permanently with some probability. This case may be applicable to public debt, but not to stock or urban real estate.

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1 Introduction

Fiat money and a rational bubble have the same property as liquidity. As shown by Samuelson (1958) and Tirole (1985), they attain a positive market value if they are expected to be exchangeable for goods in the future. It is also common that they become worthless if they are expected to be worthless in the future, given no intrinsic use of them. This property of self-fulfilling multiple equilibria has been used to explain a large boom-bust cycle in an asset price, because a stochastic transition between the two equilibria can generate a boom-bust cycle by speculation without any change in asset fundamentals.

This paper, however, casts doubt to the interpretation of a rational bubble as a non-fundamental component of the price of an interest-bearing asset in general. This paper compares fiat money and a Lucas' tree in a standard overlapping generations model. The result shows that if a Lucas' tree yields a positive dividend in each period, then it cannot have multiple equilibrium prices because an interest-bearing asset is always liquid in a frictionless competitive market. Moreover, the unique competitive equilibrium price of the Lucas' tree converges to the monetary equilibrium price of fiat money as the dividend goes to zero in the limit. These results imply that the liquidity value of a Lucas' tree is always part of the competitive market price of a Lucas' tree. Thus, the value of liquidity represented by a rational bubble cannot be interpreted as a non-fundamental component of the price of a standard interest-bearing asset.

This paper, then, discusses in what case a rational bubble can affect the price of a Lucas' tree through multiple equilibria. It is the case if there exists a positive probability that the dividend on a Lucas' tree becomes zero permanently. In this case, there can be a self-fulfilling expectation that a Lucas' tree will become a rational bubble in the perpetual state of no dividend. Such an expectation raises the price of a Lucas' tree ex-ante while it yields a positive dividend. This effect of a rational bubble, however, unlikely explains the pricing of stock or urban real estate, because stock cannot be traded after a firm goes bankrupt and because the probability that urban real estate becomes useless land is infinitesimal ex-ante. Instead, this type of a rational bubble may occur to public debt, as a government can roll over its debt perpetually. Thus, it is important to specify the type of asset to be analyzed when applying the theory of a rational bubble to an asset price.

Just to clarify, Kocherlakota (2008) introduces a constraint that requires a representative agent to hold an interest-bearing asset at an infinitely distant future date, and finds that such a constraint raises the competitive equilibrium price of the asset. He calls this increase in the asset price a rational bubble, as it represents a need for an asset as liquidity. This price movement, however, is not due to self-fulfilling expectations, but due to a change in the environment. This mechanism of an asset-price movement is different from the rational bubble analyzed in this paper.

The remainder of this paper is organized as follows. An overlapping generations model with fiat money is described in section 2. It is compared with an overlapping generations model with a Lucas' tree in section 3. A sufficient condition for the existence of multiple equilibria with a Lucas' tree is described in section 4. Section 5 concludes.

2 Overlapping generations model with fiat money

Time is discrete and indexed by $t = 0, 1, 2, ...$ In each period, a unit continuum of agents are born and live for two periods. Call agents in their first period "young", and those in their second period "old".

Each agent maximizes the following utility function:

$$U = \ln c_{1,t} + \beta \ln c_{2,t+1}, \tag{1}$$

where $\beta \in (0, 1)$, and $c_{1,t}$ and $c_{2,t}$ denote the consumption of goods by a young and an old agent, respectively, in period t. Each agent is endowed with amounts e_Y and e_O of perishable goods when young and old, respectively. Assume that

$$\beta e_Y > e_O > 0. \tag{2}$$

There also exist a unit continuum of the initial old in period 0 who maximize their consumption in the period. They exit from the economy after the consumption in period 0.

The initial old do not have any good, but are endowed with a unit amount of intrinsically useless fiat money for each. Fiat money is divisible and never depreciates. There exists a competitive market for fiat money in which agents take as given the price of fiat money in terms of goods in each period.

The maximization problem for a young agent born in period t is specified as:

$$\max_{c_{1,t}, c_{2,t+1}, m_t} \ln c_{1,t} + \beta \ln c_{2,t+1}, \tag{3}$$

$$\text{s.t. } c_{1,t} = e_Y - p_t m_t, \tag{4}$$

$$c_{2,t+1} = e_O + p_{t+1} m_t, \tag{5}$$

$$c_{1,t}, c_{2,t+1}, m_t \geq 0, \tag{6}$$

where m_t is the amount of fiat money held by a young agent at the end of period t and p_t denotes the competitive price of fiat money in terms of goods. The initial old's consumption of goods in period 0 is p_0. Given the set of parameters, $(\beta,\ e_Y,\ e_O)$, an equilibrium is characterized by the solution to the utility maximization problem for each agent and the value of p_t satisfying the market clearing condition,

$$m_t = 1, \tag{7}$$

for $t = 0, 1, 2....$

2.1 Equilibrium value of fiat money

As well known in the literature, there exist two stationary equilibria in this model under assumption (2). One is a non-monetary equilibrium, in which $p_t = 0$ for all t. The other is a monetary equilibrium in which $p_t > 0$ for all t. In the latter equilibrium, the first-order condition for m_t implies

$$p_t = \frac{\beta c_{1,t} p_{t+1}}{c_{2,t+1}}. \tag{8}$$

Substituting (4), (5), and (7) into (8) yield

$$p^* = \frac{\beta e_Y - e_O}{1 + \beta}, \tag{9}$$

where p^* denotes the price of fiat money in the stationary monetary equilibrium. Assumption (2) ensures that the right-hand side of (9) is positive.

The positive value of fiat money in the monetary equilibrium is a rational bubble, because there is no intrinsic use of fiat money. Thus, a rational bubble represents the value of liquidity that facilitates intertemporal exchange of goods.

3 Overlapping generations model with a Lucas' tree

Now substitute fiat money with a Lucas' tree. The tree yields an amount d (> 0) of goods as a dividend at the beginning of each period. Like fiat money, the tree is divisible and never depreciates. There exists a competitive market for the shares of the tree in each period. The initial old hold a unit supply of the tree in period 0.

With these new assumptions, the maximization problem for a young agent born in period t is modified to:

$$\max_{c_{1,t}, c_{2,t+1}, a_t} \quad \ln c_{1,t} + \beta \ln c_{2,t+1}, \tag{10}$$

$$\text{s.t. } c_{1,t} = e_Y - q_t a_t, \tag{11}$$

$$c_{2,t+1} = e_O + (q_{t+1} + d) a_t, \tag{12}$$

$$c_{1,t}, c_{2,t+1}, a_t \geq 0, \tag{13}$$

where q_t denotes the price of the shares of the tree in terms of goods in period t, and a_t denotes the share of the tree held by a young agent at the end of period t. The initial old's consumption of goods in period 0 is $q_0 + d$. Given the set of parameters, $(\beta,\ e_Y,\ e_O,\ d)$, an equilibrium is characterized by the solution for the utility maximization problem for each agent and the value of q_t satisfying the market clearing condition,

$$a_t = 1, \tag{14}$$

for $t = 0, 1, 2....$

3.1 Equilibrium price of a Lucas' tree

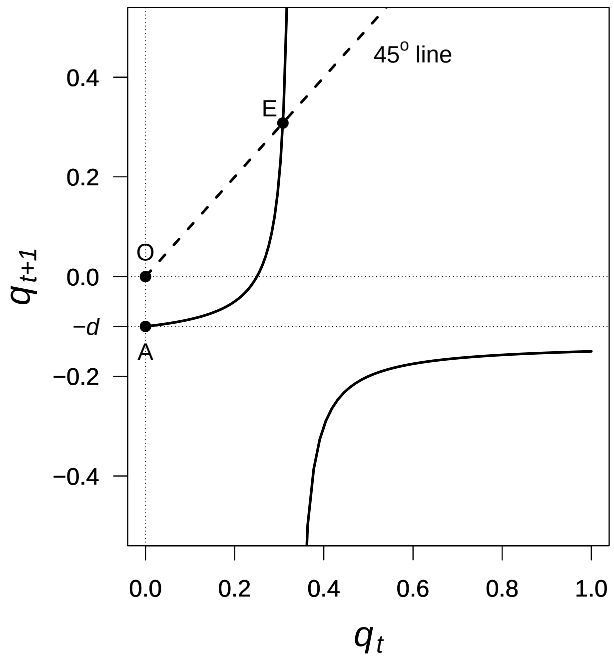
In equilibrium,

$$q_t > 0 \quad \text{for all } t, \tag{15}$$

as otherwise agents would demand an infinite share of the tree. The first-order condition for a_t is

$$q_t = \frac{\beta c_{1,t}(q_{t+1} + d)}{c_{2,t+1}}. \tag{16}$$

Figure 1: Phase diagram for the model with a Lucas' tree



Note: $(\beta,\ e_Y,\ e_O,\ d) = (0.5,\ 1,\ 0.05,\ 0.1)$. The solid curve and the dotted line are (17) and a 45-degree line, respectively.

Then, substituting (11), (12), and (14) into (16) yields

$$q_{t+1} = \frac{e_O q_t}{\beta e_Y - (1+\beta) q_t} - d, \tag{17}$$

for $t = 0, 1, 2...$

Figure 1 draws a numerical example of the phase diagram implied by (17) with a 45-degree line. The point E, i.e., $q_t = q_{t+1}$ for all t, is the unique equilibrium, because q_t must be negative in some period t to satisfy (17) if q_0 does not take the stationary equilibrium value. Even though the figure is a numerical example, this result of unique equilibrium holds for any parameter values.[1]

[1]Equation (17) is a hyperbolic function of q_t. For the domain $[0, \beta e_Y/(1+\beta))$, (17) is monotonically increasing and strictly convex over a range $[-d, \infty)$. For $q_t \in \mathbf{R} \backslash [0, \beta e_Y/(1+\beta))$, q_{t+1} takes a negative value below $-d$.

3.2 No rational bubble on a Lucas' tree with a positive dividend in each period

The uniqueness of equilibrium with a Lucas' tree contrasts with the existence of multiple equilibria with fiat money. This contrast is due to a bifurcation. The solid curve in Figure 1 moves upward as the dividend on a Lucas' tree, d, declines towards zero. In the limit, the point A converges to the origin at the point O. This limit case is the economy with fiat money; thus, the point O represents the non-monetary equilibrium. As the point A converges to the point O, the point E converges to a point on the 45-degree line with $q_t > 0$, which is the stationary monetary equilibrium with fiat money. Hence:

$$\lim_{d \searrow 0} q = p^*, \tag{18}$$

where q denotes the value of q_t and q_{t+1} at the point E.

This result implies that for $q_t \geq 0$, the solid curve and the 45-degree line in Figure 1 intersect twice if and only if $d = 0$, but only once if $d > 0$. This bifurcation property of the economy obtains because as long as d is positive, a Lucas' tree is always liquid in a frictionless competitive market. Thus, there is no other equilibrium in which a Lucas' tree loses liquidity, like fiat money does in the non-monetary equilibrium. This uniqueness of equilibrium implies that there can be no rational bubble on a Lucas' tree with a positive dividend in each period.

3.3 Decomposition of the competitive equilibrium price of a Lucas' tree

Perhaps a possible counterargument to this result is that a rational bubble represents a non-fundamental component of an asset price due to speculation. In this interpretation, a rational bubble can be added to the fundamental price of an asset to compute the total asset price. But it is difficult to adopt this interpretation. To see why, first confirm that

$$\frac{\partial q}{\partial d} > 0. \tag{19}$$

The proof for this result is straightforward: the solid curve in Figure 1 moves downward as d increases, as implied by (17); thus, the point E moves up along the 45-degree line. Then (18) and (19) jointly imply that

$$q > p^* \quad \text{for } d > 0. \tag{20}$$

This inequality indicates that the value of liquidity represented by a rational bubble and fiat money, p^*, is always part of the competitive market price of a Lucas' tree, q.

To further clarify this point, decompose q using (8) and (16):

$$\begin{aligned} q - p^* &= \Lambda(q + d) - \Lambda^* p^*, \\ &= \Lambda(q + d - p^*) - (\Lambda^* - \Lambda)p^*, \\ &= \frac{\Lambda d - (\Lambda^* - \Lambda)p^*}{1 - \Lambda}, \end{aligned} \tag{21}$$

where Λ and Λ^* denote the discount factors for the young in each period, $\beta c_{1,t}/c_{2,t+1}$, in the unique equilibrium with a Lucas' tree and the stationary monetary equilibrium with fiat money, respectively:

$$\Lambda \equiv \frac{\beta(e_Y - q)}{e_O + q + d}, \tag{22}$$

$$\Lambda^* \equiv \frac{\beta(e_Y - p^*)}{e_O + p^*}. \tag{23}$$

Rearranging both sides of (21) yields

$$\rho q = d + (1 + \rho)\frac{\rho^* p^*}{1 + \rho^*}, \tag{24}$$

where ρ and ρ^* denote the time preference rates implied by Λ and Λ^*, respectively:

$$\rho \equiv \frac{1 - \Lambda}{\Lambda}, \tag{25}$$

$$\rho^* \equiv \frac{1 - \Lambda^*}{\Lambda^*}. \tag{26}$$

In (24), the left-hand side is the effective ex-post dividend on a Lucas' tree. On the right-hand side, the first term, d, is the dividend on a Lucas' tree in the next period. The second term is the liquidity value of a Lucas' tree in the next period. This term consists of two components. The first one is the present discounted value of the effective dividend on fiat money in the next period, $\rho^* p^*/(1+\rho^*)$, in the stationary monetary equilibrium. This component represents the value of pure liquidity, which is independent of the dividend on a Lucas' tree, d. This value is multiplied by the time preference rate in the economy with a Lucas tree, $1+\rho$, to convert the presented discounted value of pure liquidity into an ex-post value in the next period.

The decomposition in (24) clarifies that the competitive market price of a Lucas' tree, q, always contains the values of dividends and liquidity. Thus, if a rational bubble is taken as a non-fundamental component of the price of a standard interest-bearing asset, then it double-counts the value of liquidity that is already part of the competitive market price of the asset.

4 In what case a rational bubble affects the price of an interest-bearing asset?

The result described so far shows that the theory of a rational bubble is not applicable to a standard interest-bearing asset. Is there any case in which a rational bubble affects the price of an interest-bearing asset through multiple equilibria? This section presents an example of such a case to demonstrate that the applicability of a rational bubble depends on the characteristics of each interest-bearing asset.

Introduce the following assumption into the model with a Lucas' tree. Suppose there are two states, F and M. The dividend on a Lucas' tree remains d (> 0) in state F and becomes zero in state M. The transition between the two states follows a Markov process: state M is the absorbing state; and the transition probability from state F to M is λ $(\in (0,1))$. Once the economy enters into state M, it is equivalent to the economy with fiat money described in section 2. Assume that the equilibrium in state M is either the non-monetary or the stationary monetary equilibrium.

Given these assumptions, the maximization problem for a young agent in state F is specified as:

$$\max_{c_{1,t}^F,\, a_t^F} \ \ln c_{1,t}^F + \beta E_t \ln c_{2,t+1}, \tag{27}$$

$$\text{s.t. } c_{1,t}^F = e_Y - q_t^F a_t^F, \tag{28}$$

$$c_{2,t+1} = \begin{cases} c_{2,t+1}^F = e_O + (q_{t+1}^F + d)a_t^F, & \text{if } S_{t+1} = F, \\ c_{2,t+1}^M = e_O + q_{t+1}^M a_t^F, & \text{if } S_{t+1} = M, \end{cases} \tag{29}$$

$$c_{1,t}^F,\ a_t^F \geq 0, \tag{30}$$

where: the superscripts F and M denote the variables for states F and M, respectively; and S_t denotes the state of the economy in period t.

The first-order condition for the share of a Lucas' tree held by a young agent in state F, a_t^F, implies that

$$\frac{q_t^F}{c_{1,t}^F} = \beta \left[\lambda \frac{q_{t+1}^F + d}{c_{2,t+1}^F} + (1-\lambda) \frac{q_{t+1}^M}{c_{2,t+1}^M} \right]. \tag{31}$$

Because either the non-monetary or the stationary monetary equilibrium prevails in state M,

$$\frac{q_{t+1}^M}{c_{2,t+1}^M} = \frac{q^M}{c_2^M}, \tag{32}$$

in equilibrium, where

$$\frac{q^M}{c_2^M} \equiv \begin{cases} \frac{p^*}{e_O + p^*}, & \text{if the stationary monetary equilibrium occurs in state } M, \\ 0, & \text{if the non-monetary equilibrium occurs in state } M. \end{cases} \tag{33}$$

Also, substituting (14) into (28) and (29) yields

$$q_t^F = e_Y - c_{1,t}^F, \tag{34}$$

$$c_{2,t+1}^F = e_Y + e_O + d - c_{1,t+1}^F. \tag{35}$$

Equations (31), (34), and (35) jointly imply that:

$$\frac{1}{c_{1,t}^F} = \frac{1}{e_Y} \left\{ 1 + \beta \left[\lambda \frac{e_Y + d - c_{1,t+1}^F}{e_Y + e_O + d - c_{1,t+1}^F} + (1-\lambda) \frac{q^M}{c_2^M} \right] \right\}. \tag{36}$$

Drawing the phase diagram implied by (36), it can be shown that (36) has only one fixed point that satisfies $c^F_{1,t} = c^F_{1,t+1} \leq e_Y$. This fixed point is the only equilibrium for a given value of q^M/c^M_2.[2] Thus, the multiple equilibrium values of q^M/c^M_2 cause multiple equilibria on the value of $c^F_{1,t}$, and hence q^F_t.

In the comparison between the two equilibria, q^F_t is higher if the stationary monetary equilibrium occurs in state M. The intuition for this result is simple: a Lucas' tree has a higher price in state F if it is expected to remain as liquidity even when it stops generating a dividend in state M.

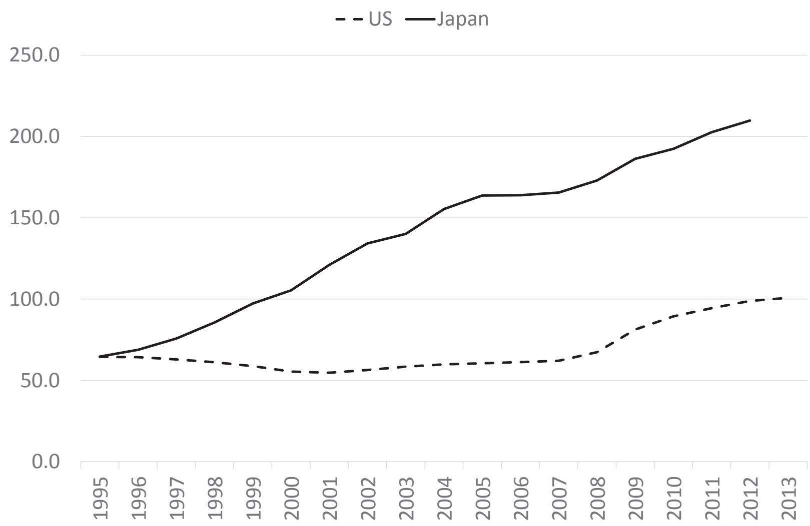
To test the applicability of this case to an interest-bearing asset, it is necessary to check if the asset in question can be traded even if it stops yielding a dividend or interest. Listed shares of a firm are unlikely to pass this test, as they disappears, and hence cannot be traded, after a firm goes bankrupt. Regarding real estate, it is possible that people trade a piece of land in a remote area solely as liquidity without any expectation of productive use of the land. But such a consideration, if any, should not affect urban real estate prices significantly, as the probability that an urban area turns into useless land is infinitesimal ex-ante.

A possible exception is public debt. Even though a rational bubble cannot occur to debt with a finite maturity, the government can issue an implicit consol bond by rolling over the principal forever. If it also rolls over interest payment perpetually, then it is effectively insolvent.[3] Indeed, as shown in Figure 2, the government debt-to-GDP ratio surpasses 100% in the U.S. and 200% in Japan. Yet, there is no hike in the long-term Treasury bond rate in these countries. This observation may indicate that government debt in these countries contains a rational bubble: people do not care about the solvency of the government as they expect public debt to remain as liquidity in any case.

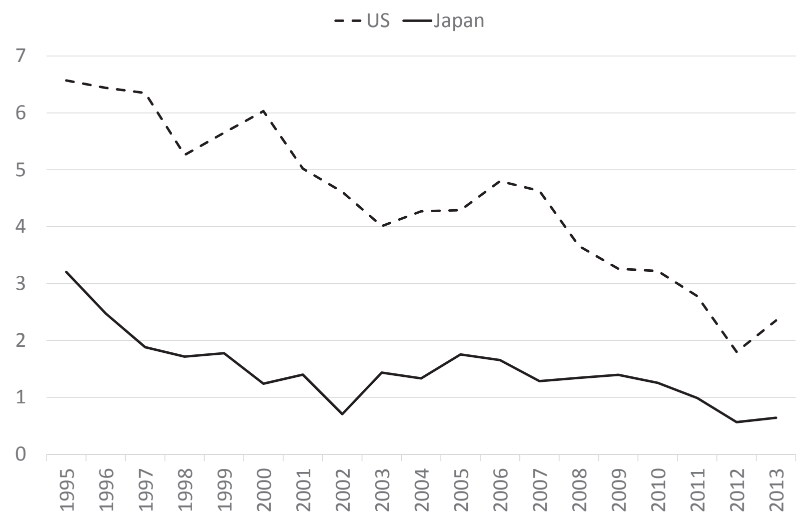
[2] Note that $c^F_{1,t} \leq e_Y$ must be satisfied for all t in equilibrium, given (28) and (30). The right-hand side of (36) is a hyperbolic function of $1/c^F_{1,t+1}$. Denote this function by $g(\cdot)$, and $1/c^F_{1,t}$ by y_t. For $y_{t+1} \geq 1/e_Y$, the function g is concave. Also, $g(1/e_Y) > 1/e_Y$. Thus, the locus of the function $y_t = g(y_{t+1})$ intersects with the 45-degree line only once for $y_{t+1} \geq 1/e_Y$. Denote the value of y_t and y_{t+1} at this fixed point by y^*. Using the phase diagram implied by the function g, it can be shown that if $y_t \neq y^*$, then $y_{t+s} < 1/e_Y$ for some positive integer s. Hence $y_t = y^*$ for all t is the only equilibrium for a given value of q^M/c^M_2.

[3] In such a case, the nominal balance of public debt keeps rising. To keep the real value of public debt stationary, the price level must be rising in the equilibrium. This effect of public debt is different from the fiscal theory of price level (e.g., Woodford 1995), as the government is insolvent in this case.

Figure 2: Public debt in Japan and the U.S.



(a) National government debt outstanding / GDP (%)



(b) 10-year Treasury bond rate

5 Conclusions

In summary, a Lucas' tree with a positive dividend in each period cannot have multiple competitive equilibrium prices, because it is necessarily liquid in a frictionless competitive market. Thus, the value of liquidity represented by a rational bubble is always part of the competitive market price of a Lucas' tree. This result implies that a rational bubble cannot be interpreted as a non-fundamental component of the price of a standard interest-bearing asset.

A rational bubble can affect the price of a Lucas' tree through multiple equilibria if there exists a positive probability that the dividend on a Lucas' tree becomes zero permanently. This special case may be applicable to public debt, but not to stock or urban real estate. This result demonstrates the importance of specifying the type of asset to be analyzed when applying the theory of a rational bubble to an asset price.

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