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Liquidity Trap and Optimal Monetary Policy Revisited *

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Abstract

This paper investigates *history dependent easing* known as a conventional wisdom of optimal monetary policy in a liquidity trap. We show that, in an economy where the rate of inflation exhibits intrinsic persistence, monetary tightening is earlier as inflation becomes more persistent. This property is referred as *early tightening* and in the case of a higher degree of inflation persistence, a central bank implements *front-loaded tightening* so that it terminates the zero interest rate policy even before the natural rate of interest turns positive. As a prominent feature in a liquidity trap, a *forward guidance* of smoothing the change in inflation rates contributes to an early termination of the zero interest rate policy.

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1 Introduction

The theory of monetary policy has been developed since 1990s based on a new Keynesian model as represented by Clarida et al. (1999) and Woodford (2003). In particular, Woodford (2003) finds history dependence as a general property of optimal monetary policy. The optimal monetary policy rule explicitly includes lagged endogenous variables and the current monetary policy reflects the past economic environment.

Among the prominent extensions, Eggertsson and Woodford (2003a,b) and Jung et al. (2001, 2005) first show optimal monetary policy in a liquidity trap in a purely forward-looking new Keynesian model. Many papers advance analyses of the optimal zero interest rate policy and their robust conclusion about a feature of optimal monetary policy is history dependence. In a liquidity trap, a central bank needs to tenaciously continue the zero interest rate policy even after the natural rate of interest turns positive. The future high inflation created by committing to continuing the zero interest rate can reduce the real interest rate, which stimulates the economy even when the current nominal interest rate is bound at zero.

The previous literature, however, is based on a forward-looking model. Woodford (2003) shows that the forward-looking economy and history dependence are two sides of a coin in optimal monetary policy. The question is: *is history dependence always a sufficient property of optimal monetary policy in a liquidity trap even when the economy has intrinsic persistence?* This question is relevant from the perspective in theory as well as in practice since an exit strategy from a zero interest rate policy can be different from a conventional wisdom.

Empirical studies using U.S. economic data show that the inflation rate is highly persistent and the Phillips curve is both forward-looking and backward-looking. Fuhrer and Moore (1995) and Galí and Gertler (1999) broadly show that a hybrid Phillips curve rather than a purely forward-looking Phillips curve is suitable for monetary policy analyses. Christiano et al. (2005) and Smets and Wouters (2007) estimate the hybrid Phillips curve in a dynamic stochastic general equilibrium model and it suits the U.S. economy.[1] Relaxing the assumption of a purely forward-looking economy is a key to illustrate the role of history dependence and new features of optimal monetary policy in a liquidity trap. To that end, using a more realistic model, i.e., a model with inflation persistence would be useful.

In this paper, we show optimal monetary policy in a liquidity trap using a standard new Keynesian model with inflation persistence. We analytically derive optimal monetary policy and investigate its features. The novel feature is that a central bank should implement early tightening rather than history dependent easing. A forward guidance of committing to smoothing the change in inflation rates contributes to this feature. In an economy with inflation persistence, the central bank's objective changes from achieving a target level of an inflation rate to smoothing the change in inflation rates. Therefore, private agents expect an accommodative monetary policy. This produces an acceleration of inflation rates and terminating the zero interest rate policy is earlier compared to the case of an economy without inflation persistence.

We examine these mechanisms by numerical simulations of when to exit from the zero interest rate policy. The optimal timing of ending the zero interest rate policy becomes earlier, as inflation persistence becomes larger. In the case of a higher degree of inflation persistence, the zero interest rate policy is terminated even while the natural rate is below zero, that is, monetary tightening is front-loaded. We also observe such a front-loaded tightening against the peak inflation rate. A strong power of forward guidance and inflation inertia contribute to the outcomes. The results are in stark contrast to those in a purely forward-looking economy.

Our paper is related to three strands of previous literature, but in stark contrast with them in the following ways. First, our paper is related to optimal monetary polciy in the model with inflation persistence such as in Woodford (2003) and Steinsson (2003).

[1] Schorfheide (2008) surveys degrees of inflation persistence through lagged inflation rates in various dynamic stochastic general equilibrium models.

In particular, Woodford (2003) derives the Phillips curve including inflation inertia by the indexation rule. Inflation inertia through indexation is realistic in a low inflation era, in particular for periods of exit from a zero interest rate policy. Terminating the zero interest rate policy requires an environment of extensive progress of inflation rates including backward-looking adjustment by indexation. Our paper clearly differs from these two papers in that we consider the zero lower bound on nominal interest rates.

Second, our paper is related to optimal monetary policy in a liquidity trap. Eggertsson and Woodford (2003a,b) and Jung et al. (2001, 2005) show that the optimal commitment policy is history dependent so that a central bank continues a zero interest rate policy even after the natural rate turns positive.[2] Adam and Billi (2006, 2007) and Nakov (2008) solve the optimal commitment policy as well as the discretionary policy under the zero lower bound on nominal interest rates with stochastic shocks. Werning (2011) shows that the future consumption boom as well as the future high inflation play important roles to mitigate a liquidity trap. Fujiwara et al. (2013) extend the model to the open economy and show an optimal zero interest rate policy in a global liquidity trap.[3] They assume a forward-looking model and find history dependence as a robust feature of optimal monetary policy. Our paper adds a missing piece of inflation persistence on these studies and show an exit strategy from a zero interest rate policy.

Third, our paper is related to the forward guidance puzzle discussed in Del Negro et al. (2012) and McKay et al. (2015). They point out that the future forward guidance by a central bank is extremely powerful in a liquidity trap so that it drastically lifts the inflation rate and the output gap. Our result reveals that the power of forward guidance

[2]Eggertsson and Woodford (2006) and Eggertsson (2006, 2008, 2012) reveal roles of fiscal policy as well as monetary policy in a liquidity trap.

[3]There are many other influential papers regarding optimal monetary policy in a liquidity trap. For example, Jeanne and Svensson (2007) show the important role of currency depreciation and price level targeting as a commitment device to escape from a liquidity trap. Billi (2011) focuses on the optimal long-run inflation rate to preempt falling into a liquidity trap. Evans et al. (2015) show an exit strategy from the zero interest rate policy under a suboptimal policy, i.e., optimal discretionary policy, using a purely forward-looking model and a purely backward-looking model.

is strengthened in an economy which exhibits inflation persistence and a liquidity trap. The forward guidance is so strong that it offsets initial deflation in a liquidity trap. This is a distinct feature of optimal monetary policy only in an economy with inflation persistence. A reason for this is that a central bank's commitment to smoothing the change in inflation rates induces expectations for an accommodative monetary policy, which accelerates inflation rates. Furthermore, unlike the conclusion of McKay et al. (2015), the forward guidance puzzle cannot be fully solved in the case of the optimal commitment policy.

The remainder of the paper proceeds as follows. Section 2 presents a model in the economy with inflation persistence. Section 3 derives an optimal monetary policy in a liquidity trap and Section 4 examines numerical simulations to show the optimal exit strategy from a zero interest rate policy. Section 5 shows robustness and applications. Section 6 concludes.

2 The Model

We use a new Keynesian model proposed by Woodford (2003). The macroeconomic structure is expressed by the following three equations:

$$x_t = \mathrm{E}_t x_{t+1} - \chi \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n \right), \tag{1}$$

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \beta \left(\mathrm{E}_t \pi_{t+1} - \gamma \pi_t \right) + \mu_t, \tag{2}$$

$$r_t^n = \rho_r r_{t-1}^n + \epsilon_t^r, \tag{3}$$

where χ, κ, β, γ, and ρ_r are parameters, satisfying $\chi > 0$, $\kappa > 0$, $0 < \beta < 1$, $0 \leq \gamma \leq 1$, and $0 \leq \rho_r < 1$. x_t, i_t and π_t denote the output gap, the nominal interest rate (or policy rate), and the rate of inflation in period t, respectively. The expectations operator E_t covers information available in period t. r_t^n is the natural rate of interest, which is assumed to follow an AR(1) process. ϵ_t^r is i.i.d. disturbance with variances of σ_r. μ_t is the cost-push shock that is i.i.d. disturbance with variances of σ_μ.

Equation (1) is the forward-looking IS curve. The IS curve states that the current output gap is determined by the expected value of the output gap and the deviation of the current real interest rate, defined as $i_t - \mathrm{E}_t\pi_{t+1}$, from the natural rate of interest.

Equation (2) is the hybrid Phillips curve. γ denotes the degree of inflation persistence. In particular, when $\gamma = 0$, the hybrid Phillips curve collapses to a purely forward-looking Phillips curve, in which current inflation depends on expected inflation and the current output gap. When $0 < \gamma \leq 1$, the Phillips curve is both forward-looking and backward-looking and the current inflation rate depends on the lagged inflation rate as well as the expected inflation and the current output gap. As γ approaches one, the coefficient on the lagged inflation rate approaches 0.5.

In this paper, we assume inflation persistence with indexation. Specifically, we follow Woodford (2003), who derives the Phillips curve including inflation inertia with a micro-foundation.[4] In the indexation rule, some firms that cannot reoptimize their own goods prices adjust current prices based on the past inflation rate. The indexation mechanism is empirically supported by Christiano et al. (2005) and Smets and Wouters (2007). We can analyse both the purely forward-looking Phillips curve and the hybrid Phillips curve by changing parameters of inflation persistence.

Next, we consider the central bank's intertemporal optimization problem. The central bank sets the nominal interest rate i_t so as to minimize the welfare loss $\mathcal{L}_t$ defined as

$$\mathcal{L}_t = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \mathrm{L}_{t+i}, \tag{4}$$

where L_t is the period loss function obtained by second-order approximation of the household utility function. In an economy with inflation inertia, Woodford (2003) shows that L_t is given by

$$\mathrm{L}_t = (\pi_t - \gamma\pi_{t-1})^2 + \lambda_x x_t^2,$$

where λ_x is a non-negative parameter. A central bank needs to stabilize $\pi_t - \gamma\pi_{t-1}$ rather than the inflation rate itself when inflation exhibits intrinsic persistence. In an economy without inflation persistence, dispersion comes from an environment where some firms

[4]There are several theoretical foundations to introduce inflation persistence. For example, Mankiw and Reis (2002) introduce information rigidity to produce inflation persistence. Milani (2007) points out the importance of an agent's learning for inflation persistence.

reoptimize prices and other firms do not change prices at all. In an economy with indexation on inflation rates, however, dispersion comes from an environment where some firms not reoptimizing their prices follow the past inflation rate with a certain degree in their price setting and other firms reoptimize prices. Therefore, to minimize price dispersion, a central bank needs to set the current inflation rate so as to be close to the adjusted lagged inflation rate.

Finally, we impose a nonnegativity constraint on the nominal interest rate:

$$i_t \geq 0. \tag{5}$$

It should be noted that the presence of a nonnegativity constraint introduces nonlinearity in an otherwise linear-quadratic model. The central bank maximizes equation (4) subject to equations (1)-(3) and (5).

3 Optimal Monetary Policy in a Liquidity Trap

We analytically characterize optimal monetary policy in a liquidity trap and clarify an implication of an optimal exit strategy. Optimal monetary policy under the zero lower bound on the nominal interest rate in a timeless perspective is expressed by the solution of the optimization problem.[5] To investigate features of optimal monetary policy, we denote the degree of inflation persistence in the hybrid Phillips curve as γ_{pc} and that in the period loss function as γ_{loss}. This setup is just to clarify the mechanism of inflation persistence and we set $\gamma_{pc} = \gamma_{loss} = \gamma$ in the benchmark. The optimization problem is represented by the following Lagrangian form:

$$\mathcal{L} = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \begin{array}{c} \left(\pi_{t+i} - \gamma_{loss}\pi_{t+i-1}\right)^2 + \lambda_x x_{t+i}^2 \\ -2\phi_{1t+i}\left[x_{t+i+1} - \chi\left(i_{t+i} - \pi_{t+i+1} - r_{t+i}^n\right) - x_{t+i}\right] \\ -2\phi_{2t+i}\left[\kappa x_{t+i} + \beta\left(\pi_{t+i+1} - \gamma_{pc}\pi_{t+i}\right) - \pi_{t+i} + \gamma_{pc}\pi_{t+i-1}\right] \end{array} \right\},$$

[5] The central bank solves an intertemporal optimization problem in period t, considering the expectation channel of monetary policy, and commits itself to the computed optimal path. This is the optimal solution from a timeless perspective defined by Woodford (2003).

where ϕ_1 and ϕ_2 are the Lagrange multipliers associated with the IS constraint and the Phillips curve constraint, respectively. We differentiate the Lagrangian with respect to π_t, x_t, and i_t under the nonnegativity constraint on nominal interest rates to obtain the first-order conditions:

$$-\beta\gamma_{loss}\left(\mathrm{E}_t\pi_{t+1}-\gamma_{loss}\pi_t\right)+\pi_t-\gamma_{loss}\pi_{t-1}-\beta^{-1}\chi\phi_{1t-1}-\beta\gamma_{pc}\mathrm{E}_t\phi_{2t+1}+\left(\beta\gamma_{pc}+1\right)\phi_{2t}-\phi_{2t-1}=0, \tag{6}$$

$$\lambda_x x_t+\phi_{1t}-\beta^{-1}\phi_{1t-1}-\kappa\phi_{2t}=0, \tag{7}$$

$$i_t\phi_{1t}=0, \tag{8}$$

$$\phi_{1t}\geq 0, \tag{9}$$

$$i_t\geq 0. \tag{10}$$

Equations (8), (9), and (10) are conditions for the nonnegativity constraint on nominal interest rates. The above five conditions, together with the IS curve of equation (1) and the hybrid Phillips curve of equation (2), govern the loss minimization. The optimal interest rate is determined by these conditions each period. We also need initial conditions for all variables being zero except the nominal interest rate, which takes a positive value in the steady state. When the nonnegativity constraint is not binding, i.e., $i_t>0$, the Lagrange multiplier ϕ_{1t} becomes zero by the Kuhn-Tucker condition in equation (8), and the interest rate is determined by the conditions given by equations (1), (2), (6), and (7) with $\phi_{1t}=0$. When the nonnegativity constraint is binding, i.e., $i_t=0$, the interest rate is simply set to zero. The interest rate remains zero until the Lagrange multiplier ϕ_{1t} becomes zero.

We cannot solve this system using the standard solution method because of the nonnegativity constraint on nominal interest rates, and numerical simulations are required to obtain the path of variables under optimal monetary policy in a liquidity trap. The first-order conditions in period t given by equations (6) and (7), however, characterize qualitative features of optimal monetary policy in a liquidity trap and in the economy with inflation persistence.

The first feature is that, due to the central bank's objective to minimize the change in inflation rates, i.e., $\pi_t - \gamma\pi_{t-1}$, the optimality condition includes terms to smooth inflation rates as shown in equation (6). Specifically, the expected change in inflation rates as well as the current change in inflation rates induce a strong commitment to inflation smoothing. A high inflation rate comes with a high expected inflation rate. Thus, in an economy with inflation persistence, agents expect more accommodative monetary policy than in an economy where the central bank's objective is to minimize the deviation of inflation rates from a target level.

The second feature of optimal monetary policy is forward-looking terms associated with introducing inflation persistence into the model. The central bank implements monetary policy based on a forecast of future inflation rates and the output gap. There are two channels to make optimal monetary policy forward-looking. The first channel functions through the parameter γ_{loss} on the future inflation rate in equation (6). Optimal monetary policy in a model with inflation persistence should respond to the expected inflation rate. The second channel works through the parameter γ_{pc} in equation (6) on the Lagrange multiplier ϕ_{2t+1} that is related to the future output gap and a future zero interest rate condition. Note that the optimality condition includes the backward-looking variables, which induces history dependent policy in a similar vein as the standard model. Theoretically, both forward-looking and backward-looking elements contribute to determining the optimal path of the nominal interest rates, including the optimal timing of exit from the zero interest rate.

When comparing the optimal targeting rule with that in the previous literature, the features of optimal monetary policy become evident.[6]

[6] We can derive an optimal price-level targeting rule which exactly achieves the same optimal commitment solution as the inflation targeting rule. Defining a price-level $\widetilde{p}_t$ and a price-level target p_t^*

$$\begin{aligned}&\beta\gamma_{pc}\mathrm{E}_t\phi_{1t+1} - (1+\gamma_{pc}+\beta\gamma_{pc})\,\phi_{1t} + \left(1+\beta^{-1}+\gamma_{pc}+\beta^{-1}\kappa\chi\right)\phi_{1t-1} - \beta^{-1}\phi_{1t-2} \\ = \;& -\kappa\beta\gamma_{loss}\left(\mathrm{E}_t\pi_{t+1}-\gamma_{loss}\pi_t\right) + \kappa\left(\pi_t-\gamma_{loss}\pi_{t-1}\right) - \beta\lambda_x\gamma_{pc}\mathrm{E}_t\triangle x_{t+1} + \lambda_x\triangle x_t. \quad (11)\end{aligned}$$

This optimal targeting rule includes the zero interest rate condition given by ϕ_1. The optimal targeting rule is forward-looking due to inflation persistence as well as backward-looking. The change in inflation rates is directly related to optimal monetary policy. The rule reveals that the coefficient on $\pi_t - \gamma_{loss}\pi_{t-1}$ is positive, i.e., there is a negative effect on ϕ_{1t}, and the zero interest rate policy should be terminated when the inflation rate sufficiently accelerates. It, however, notes that the coefficient on $\mathrm{E}_t\pi_{t+1} - \gamma_{loss}\pi_t$ is negative, i.e., there is a positive effect on ϕ_{1t}. An acceleration of the inflation rate in the future works to keep a zero interest rate policy since a central bank has an incentive to smooth inflation rates. As a result, an acceleration of expected inflation rate induces an acceleration of the current inflation rate, which contributes to strengthening the effect of the commitment policy and increases inflation rates.[7] Therefore, the zero interest rate policy is terminated earlier.

If the nominal interest rate does not hit the zero lower bound, ϕ_1 becomes zero and the optimal targeting rule (11) can be reduced to backward-looking as shown in Woodford

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as

$$\widetilde{p}_t \equiv p_t - \gamma p_{t-1} + \frac{\lambda_x}{\kappa}x_t,$$

$$\phi_{1t} \equiv \kappa\left(p_t^* - \widetilde{p}_t\right),$$

we have the following optimal price-level targeting rule.

$$p_t^* \equiv \frac{\gamma\beta}{1+\gamma\beta}\mathrm{E}_t p_{t+1}^* + \frac{1}{1+\gamma\beta}p_{t-1}^* - \frac{\gamma}{1+\gamma\beta}Q_t + \frac{1}{1+\gamma\beta}\left(\gamma+\beta^{-1}-\frac{\kappa\chi}{\beta}\right)Q_{t-1} - \frac{\beta^{-1}}{1+\gamma\beta}Q_{t-2},$$

where $Q_t \equiv (p_t^* - \widetilde{p}_t)$ for simplicity. The prominent feature of the rule is $\mathrm{E}_t p_{t+1}^*$. The price-level target should depend on the future target level of price associated with future economic conditions. When γ is zero, this rule is reduced to the one in Eggertsson and Woodford (2003a,b).

[7]We make this point clearer in terms of the level of the inflation rate in Appendix A.

(2003).[8]

$$\kappa\left(\pi_t - \gamma_{loss}\pi_{t-1}\right) + \lambda_x \triangle x_t = 0.$$

Unlike equation (11), the rule is not hybrid, implying that forward-looking terms drop from the targeting rule. The forward guidance of smoothing inflation rates weakens since there is only one term for the change in inflation rates in the case where the nominal interest rate does not hit the zero lower bound. It is a phenomenon of a liquidity trap that strengthens the forward guidance by committing to a zero interest rate policy.

When γ is zero, this rule collapses to the standard optimal targeting rule in the forward-looking new Keynesian model as follows:

$$\kappa\pi_t + \lambda_x \triangle x_t = 0.$$

4 Optimal Exit Policy

4.1 Basic Calibration

In this section, we numerically solve the model and characterize the optimal exit strategy from the zero interest rate policy. The baseline quarterly parameters are typical for the U.S. economy as in Table 1. We set $\chi = 6.25$, $\alpha = 0.66$, and $\kappa = 0.0244$ in structural equations from Woodford (2003).[9] Based on these structural parameters, we calculate $\lambda_x = 0.048/16$. The natural rate shock is stochastic with variance $\sigma_r = 0.2445$ and persistence $\rho_r = 0.8$, as in Adam and Billi (2006). The steady state real interest rates is set to be 3.5 percent annually. The model is solved numerically by the collocation

[8]Another form is given by

$$\kappa\left(\mathrm{E}_t\pi_{t+1} - \gamma_{loss}\pi_t\right) + \lambda_x \mathrm{E}_t x_{t+1} = 0.$$

[9]Several studies estimate γ in range from about 0.2 to 1. For example, Smets and Wouters (2007) estimate γ as 0.24. Giannoni and Woodford (2004), Christiano et al. (2005), and McKay et al. (2015) imply the case of full indexation and set γ as one.

method and technical methodology to implement simulations is described in Appendix B.

Figure 1 shows optimal responses of the interest rate to natural rate shocks for different inflation inertia.[10] A central bank starts the zero interest rate policy even when the natural rate shock is still positive. This is an effect of uncertainty of shocks as pointed out in Adam and Billi (2006). Even in the presence of inflation inertia, uncertainty of the natural rate shock requires a central bank to conduct preemptive monetary easing. The additional contribution of introducing inflation persistence is that the zero interest rate policy is terminated earlier, as inflation persistence becomes larger in response to the natural rate shocks.

4.2 Simulations

4.2.1 One-time Shock

We assume a simple situation where a one-time shock with a persistence of $\rho_r = 0.8$ occurs in period 0. In particular, we give a 2 percent negative natural rate shock (equivalent to 8 percent annually) to make the economy into a liquidity trap. [11] We also give a larger shock, i.e., an annual 12 percent negative natural rate shock with a persistence of 0.8.

Figure 2 shows the timing of an optimal exit from an zero interest rate in response to an annual 2 percent negative natural rate shock for different degrees of inflation inertia. Interest rates are annualized in the figure. We observe several quantitative characteristics in the impulse responses.

As a common feature, a central bank sets the nominal interest rate at zero for the first several periods to bring overshooting of inflation rates and reduce real interest rates to stimulate the economy in any case. Afterwards, the central bank increases the nominal interest rate and the inflation rate returns to zero. This outcome is consistent with

[10]Note that Figure 1 does not show the whole feature of optimal monetary policy in the sense that other state variables are set at zero.

[11]For example, Jung et al. (2001, 2005) assume at least a 2 percent one-time negative shock to make the economy into a liquidity trap.

Eggertsson and Woodford (2003a,b) and Jung et al. (2001, 2005) that show that the zero interest rate policy continues even after the natural rate turns positive in the case of the purely forward-looking economy, i.e., $\gamma = 0$.

The distinct feature of optimal monetary policy is early tightening with inflation inertia increasing. As shown in Figure 2, when we assume $\gamma_{loss} = 0.8$, a timing to terminate the zero interest rate policy is sufficiently earlier compared to the case without inflation persistence. In an economy with inflation persistence, even in response to a negative shock, the inflation rate registers a positive number for the initial period and accelerates afterward.[12] Qualitatively, two reasons are worth being mentioned. First, the outcome results from a strong power of forward guidance by the commitment policy as shown in Del Negro et al. (2012) and McKay et al. (2015). In particular, a central bank should stabilize $\pi_t - \gamma\pi_{t-1}$ rather than the inflation rate itself in an economy with inflation persistence. Based on this behavior by the central bank, private agents expect that a current high inflation will induce a high expected inflation rate in the future, which accelerates inflation rates. Second, inflation persistence itself accelerates inflation rates in an intrinsic way as the degree of inflation inertia increases. A high inflation rate in the past contributes to increasing inflation rates in the future. These reasons contribute to an early termination of the zero interest rate policy.

To quantitatively examine how these two elements affect the inflation dynamics and the zero interest rate policy, we show a case of $\gamma_{loss} = 0$, given that other γ set to be 0.4 in Figure 3. In this case, the economy starts with initial deflation and inflation rates remain low unlike the case of all $\gamma = 0.4$. This result is similar to the one of $\gamma = 0$ in Figure 2. It reveals that the commitment to stabilizing $\pi_t - \gamma_{loss}\pi_{t-1}$ accelerates inflation rates. To identify which of the two terms of the change in inflation rates in equation (11) strengthens the effect of the commitment policy, we set only γ_{loss} of $\mathrm{E}_t\pi_{t+1} - \gamma_{loss}\pi_t$ to be zero. Then, inflation rates become sufficiently subdued compared to the case of all $\gamma = 0.4$, but remain high compared to the case of all $\gamma_{loss} = 0$. This implies that two terms quantitatively function as accelerators of inflation rates. We also show a case of

[12]Initial inflation rates can be negative for small γ such as 0.1.

setting γ_{pc} only in the hybrid Phillips curve to be zero in Figure 3. Even though a timing to end the zero interest rate policy does not change, a central bank sets the policy rate lower compared to the case of $\gamma = 0.4$. This is an effect of no inflation persistence in the Phillips curve.

Specifically, Figure 4(a) confirms that a zero interest rate policy is terminated earlier, as the persistence of inflation becomes larger. Figure 4(a-1) shows the time lag between a period when a zero interest rate policy is terminated, T_r, and a period when the natural rate becomes positive, T_{rn}, for different degrees of inflation inertia. It is shown that early tightening policy becomes stronger as inflation persistence becomes larger. In response to an annual 8 percent negative shock, the timing of terminating a zero interest rate policy is earlier by 2 quarters in the case of $\gamma = 0.8$ than that in the case of $\gamma = 0$. In the case of $\gamma = 0.8$, a central bank starts to increase the interest rate in a timing when the natural rate turns to be positive, i.e., $T_r - T_{rn} = 0$. There is no history dependent easing. Furthermore, the early tightening policy becomes more evident as the size of the negative natural rate shocks becomes larger. When $\gamma = 0.8$ and there is an annual 12 percent negative shock, a central bank ends the zero interest rate policy even while the natural rate remains negative since $T_r - T_{rn} = -1$. This is called front-loaded tightening, which is in stark contrast to history dependent easing.

In Figure 4(a-2), we investigate the time lag between a period when a zero interest rate policy is terminated and a period when the inflation rate hits its peak, T_p, since the inflation rate is one of the key variables to decide the exit from a zero interest rate policy. Figure 4(a-2) shows that the time lag between a period when a zero interest rate policy is terminated and a period when the inflation rate hits its peak becomes smaller as inflation inertia becomes larger. In response to an annual 8 percent negative shock, the timing of terminating a zero interest rate policy is earlier by 3 quarters in the case of $\gamma = 0.8$ than that in the case of $\gamma = 0$ in relation to the peak inflation rate. In the case of $\gamma = 0.8$, a central bank terminates the zero interest rate policy immediately after the inflation rate hits its peak, i.e., $T_r - T_p = 1$. This result is a new finding against Eggertsson and Woodford (2003a,b) and Jung et al. (2001, 2005) that show that a zero interest rate policy continues for certain periods even after the inflation rate hits its peak. This tendency remains unchanged for larger negative natural rate shocks.

4.2.2 Sequential Shock

Eggertsson and Woodford (2003a,b) assume that negative annual 2 percent shocks continue to occur for several years with a certain probability to produce a prolonged liquidity trap. In a similar vein, we assume a situation where negative natural rate shocks continue for a certain period, which is a realistic assumption to replicate a liquidity trap.

Figure 4(b) shows a case where annual 2 percent negative shocks with persistence of 0.8 continue to occur for 10 quarters. The results are similar to those for one-time shock. Both panels of 4(b-1) and 4(b-2) confirm that history dependence becomes weaker as inflation inertia becomes larger. Inflation persistence induces a nontrivial implication for the optimal exit from the zero interest rate. The timing to terminate a zero interest rate policy is earlier by 4 quarters in the case of $\gamma = 0.8$ than that in the case of $\gamma = 0$ in relation to the natural rate of interest and the peak inflation rate. With a high degree of inflation persistence, a central bank increases its policy rate even before the natural rate returns to zero, shown as $T_r - T_{rn} = -2$. This shows the case where optimal monetary policy implements the front-loaded tightening. Moreover, in the case of $\gamma = 0.8$, the zero interest rate policy is terminated immediately after the inflation rate hits its peak, i.e., $T_r - T_p = 1$.

Even if we assume a different sequential shock, i.e., annual 4 percent negative shocks with persistence of 0.8 continue to occur for 4 quarters, we can draw the same conclusion that early tightening becomes more pronounced as inflation inertia becomes stronger.

5 Robustness Analyses and Applications

For robustness analyses and applications, we show four analyses: introducing deflationary shock, weakening forward guidance, analysing a simple price-level targeting rule and an optimal interest rate rule.[13]

5.1 Deflation and Cost-push Shocks

In Figure 2 and 4, we cannot observe a clear deflation. In the actual economy, however, inflation rates could deviate below the target of inflation rates. This prompts a question as to how the economy behaves when the economy starts with deflation. To that end, we assume a negative cost-push shock in the equation of the hybrid Phillips curve. Following Adam and Billi (2006), by adding a temporary cost-push shock with $\sigma_\mu = 0.154$ and no persistence as well as a natural rate shock, we obtain the optimal response functions.

Figure 5 shows the impulse responses to an annual 8 percent one-time negative natural rate shock with a persistence of 0.8 and annual 2 percent negative cost-push shocks continuing for 5 quarters. The combination of the two negative shocks produces deflation for the first several periods. Even in the case where the economy starts with deflation, however, an early tightening policy is optimal. In particular, this characteristic becomes more pronounced in relation to the inflation rate as shown in Figure 6(a) when inflation inertia becomes larger. With the large degree of inflation persistence, the inflation rate hits its peak sufficiently after a central bank begins to raise the policy rate, i.e., $T_r - T_p = -2$ in the case of $\gamma = 0.8$.

Moreover, for an annual 12 percent one-time negative natural rate shock with a persistence of 0.8 and annual 2 percent negative cost-push shocks continuing for 5 quarters, $T_r - T_{rn} = -1$ in the case of $\gamma = 0.8$, which confirms front-loaded tightening in the conduct of optimal policy.

5.2 Weakening Forward Guidance

Del Negro et al. (2012) and McKay et al. (2015) point out that forward guidance by the commitment policy is extremely powerful in a liquidity trap in that it drastically raises the inflation rate and the output gap, which is called the forward guidance puzzle. As

[13]Regarding additional analyses for a case of low elasticity of the output gap to real interest rates and a case of wage indexation, see Appendix D. Also see Sugo and Teranishi (2008) for additional analyses.

shown in previous sections, one of our contributions is that in an economy with inflation persistence, the power of forward guidance becomes stronger than one without inflation persistence.

To solve the forward guidance puzzle, we assume a discounted IS curve in McKay et al. (2015) as follows.

$$x_t = \delta \mathrm{E}_t x_{t+1} - \chi \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n \right).$$

The discounted IS curve is different from the standard one since a discounting parameter of δ is multiplied to the expected output gap. The effects of future real interest rates are discounted, and the forward guidance should be less powerful. The first-order condition of equation (7) is replaced by[14]

$$\lambda_x x_t + \phi_{1t} - \delta \beta^{-1} \phi_{1t-1} - \kappa \phi_{2t} = 0,$$

In the numerical simulation to assess the effect of the discounted model, we set δ to be 0.8, with other parameters unchanged. Compared to McKay et al. (2015) assuming $\delta = 0.97$, a smaller value for δ is necessary to make a clear difference from the results with the standard IS curve.

Figure 7 shows impulse responses to an annual 8 percent one-time negative natural rate shock with a persistence of 0.8 when $\gamma = 0.4$. We observe a deeper recession and lower inflation rates, especially for the first few periods in the case of the discounted IS curve. This results from reducing the power of forward guidance. On the other hand, inflation rates accelerate after the initial first few periods due to inflation inertia and a lower power of the forward guidance in monetary tightening.

Figure 6(a) shows $T_r - T_{rn}$ and $T_r - T_p$ in the case of the discounted IS curve for annual 8 and 12 percent one-time negative natural rate shocks with persistence of 0.8. The outcome does not change even when the forward guidance puzzle is mitigated, that

[14] In the discounted IS curve model, the utility-based loss function consists of $(\pi_t - \gamma \pi_{t-1})^2$ and x_t^2 in a similar way as our model in Section 2, though weight parameters for these two elements in the loss function should change.

is, a zero interest rate policy is terminated earlier, as the persistence of inflation becomes larger.

5.3 Failure of Simple Price-Level Targeting Rule

Several papers, such as Eggertsson and Woodford (2003a) in a closed economy and Fujiwara et al. (2013) in an open economy, reveal that a price-level targeting rule can mitigate a liquidity trap and improve social welfare since the rule resembles the commitment policy and generates history dependent easing. The outcomes of our paper raise a question of whether a price-level targeting rule still replicate features of the optimal commitment policy in an economy with inflation persistence. A price-level targeting rule committing to monetary easing to raise inflation to a high level can strengthen forward guidance and induce an early tightening. Following Fujiwara et al. (2013), we assume a following price-level targeting rule instead of optimal commitment policy in the simulation.

$$i_t = Max\left(0,\ \hat{\imath}_t^p\right),$$

$$\hat{\imath}_t^p = \eta_p\left(\ln P_t - \ln P^*\right),$$

where $\eta_p = 5$ and $\ln P^*$ is the steady state value of a price level $\ln P_t$.

In Figure 8 that shows impulse responses to an annual 8 percent one-time negative natural rate shock with a persistence of 0.8, we observe intuitive but totally different outcomes from a common feature of optimal monetary policy in an economy with inflation persistence. The periods of zero interest rate policy become longer as inflation persistence becomes larger according to the dynamics of the price level. Specifically, Figure 6(b) shows the time difference of $T_r - T_{rn}$ and $T_r - T_p$ for different degrees of inflation inertia against annual 8 and 12 percent one-time negative natural rate shocks with persistence of 0.8. As shown in Figure 6(b-1), the timing of terminating a zero interest rate policy is later by 1 quarter in the case of $\gamma = 0.8$ than that in the case of $\gamma = 0$.[15] A price-level

[15]This result is robust to changes in the forms of price-level targeting rules such as:

$$\hat{\imath}_t^p = \eta_p\left(\ln P_t - \ln P^*\right) + \eta_x x_t.$$

targeting rule requires more history dependent easing when the natural rate shocks have more persistent negative effects on the economy through a lag in the inflation rate. A price-level targeting rule does not share the feature with optimal monetary policy in an economy with inflation persistence. This tendency is more evident for larger negative natural rate shocks.

5.4 Optimal Interest Rate Rule in a Liquidity Trap

We have considered a targeting rule to capture the property of the model. We can derive an optimal interest rate rule as well. The prominent feature of the optimal interest rate rule is that we directly observe the one-to-one relationship between the nominal interest rate and the future, current, and past endogenous variables. As shown in Giannoni and Woodford (2003) and Giannoni (2014), by introducing real balance into one of the arguments of the household's utility function, the second-order approximation of the period loss function is given by

$$\mathrm{L}_t = \left(\pi_t - \gamma \pi_{t-1}\right)^2 + \lambda_x x_t^2 + \lambda_i \left(i_t - i^*\right)^2,$$

where λ_i is nonnegative parameter and i^* denotes the steady state interest rate. We set $\lambda_i = 0.077$ from Woodford (2003) and $i^* = 0.875$ from a value of the steady state real interest rates. The first two terms are the same as the targeting rule model, while the third term represents the central bank's desire for interest rate stability. From the viewpoint of the stability for interest rate, a central bank hesitates to adopt the zero interest rate policy.

By solving the model using the modified loss function, we can derive a generalized optimal interest rate rule, which is given by[16]

$$i_t = Max\left(0,\ \hat{\imath}_t\right),$$

For $5 \leq \eta_p \leq 10$ and $0 \leq \eta_x \leq 0.5$, we have similar results.

[16]See Appendix C for the proof.

$$\mathrm{E}_t \left\{ \psi_1 \left(1-\psi_2 L\right)\left(1-\psi_3 L\right)\left(1-\psi_4 F\right)\left(\hat{\imath}_t - i^*\right) \right\} = \left\{ \begin{array}{c} \phi_\pi^* \left[-\beta\gamma \mathrm{E}_t \pi_{t+1} + \left(\beta\gamma^2+1\right)\pi_t - \gamma\pi_{t-1} \right] \\ +\phi_x^* \left[-\beta\gamma \mathrm{E}_t x_{t+1} + \left(\beta\gamma+1\right) x_t - x_{t-1} \right] \end{array} \right\}, \tag{12}$$

where $\psi_1\psi_2\psi_3 = \beta^{-2}\gamma^{-1}$, $\psi_1\psi_2 + \psi_1\psi_3 + \psi_2\psi_3 = (\beta\gamma)^{-1}\left[1+\beta^{-1}\left(1+\beta\gamma+\kappa\chi\right)\right]$, $\psi_1 + \psi_2 + \psi_3 = (\beta\gamma)^{-1}\left(1+\beta\gamma+\gamma\right)$, $\psi_4 = \psi_1^{-1}$ $(\psi_2 > \psi_3)$, $\phi_\pi^* \equiv \kappa\chi\left(\beta\gamma\lambda_i\right)^{-1}$, and $\phi_x^* \equiv \chi\lambda_x\left(\beta\gamma\lambda_i\right)^{-1}$. L and F denote the lag and forward operators, respectively.

Note that i_t cannot take a negative value, while $\hat{\imath}_t$ can. $\hat{\imath}_t$ is interpreted as an indicator variable that provides the information necessary to implement optimal monetary policy under the zero lower bound on the nominal interest rate.

The monetary policy rule given by equation (12) includes both forward-looking and backward-looking terms to determine the current value of $\hat{\imath}_t - i^*$ and the nominal interest rate. The lagged inflation rates in the loss function and in the hybrid Phillips curve induce forward-looking terms in the rule as shown in equation (6).[17]

Without inflation inertia, i.e., $\gamma = 0$, the generalized optimal interest rate rule is reduced to

$$i_t = Max\left(0,\ \hat{\imath}_t\right),$$

$$\left(1-\psi_5 L\right)\left(1-\psi_6 L\right)\left(\hat{\imath}_t - i^*\right) = \phi_\pi \pi_t + \phi_x\left(x_t - x_{t-1}\right).$$

where $\psi_5 + \psi_6 = 1 + \beta^{-1} + \beta^{-1}\kappa\chi$, $\psi_5\psi_6 = \beta^{-1}$, $\phi_\pi \equiv \kappa\chi\left(\lambda_i\right)^{-1}$, and $\phi_x \equiv \chi\lambda_x\left(\lambda_i\right)^{-1}$. The optimal interest rate rule given $\gamma = 0$ includes only backward-looking terms of $\hat{\imath}_t - i^*$ and x_t, which induce history dependence. A central bank following the interest rate rule seeks to prolong zero interest rate policy.

Figure 6(b) shows the time difference of $T_r - T_{rn}$ and $T_r - T_p$ for different degrees of inflation inertia against annual 8 and 12 percent one-time negative natural rate shocks

[17] Equation (12) is a generalization of the optimal interest rate rules shown in Giannoni and Woodford (2003) that does not consider a nonnegativity constraint on the nominal interest rate. The rule given by equation (12) achieves the same equilibrium as the Giannoni–Woodford rules when the zero lower bound on the nominal interest rate does not bind.

with persistence of 0.8. We affirm that even in the case of following interest rate rule early tightening can be attained for the sufficient large inflation persistence for any size of shocks. The interest rate rule shows front-loaded tightening for several cases in which the central bank cares about being penalized on a deviation of the interest rate from its steady state.

6 Concluding Remarks

If the economy exhibits inflation persistence and a liquidity trap, optimal monetary policy is different from conventional wisdom, that is, history dependence is not a sufficient condition for optimality.

The outcomes of this paper show that front-loaded tightening rather than the history dependent easing is dominant in a liquidity trap when inflation exhibits a higher degree of inflation persistence. The central bank should not wait for observed economic overheating in escaping from a liquidity trap with inflation persistence. The central bank ends the zero interest rate policy even while the natural rate is below zero and even before the inflation rate hits its peak.

We consider only the commitment policy in this paper and the natural extension is to check how the economy behaves when the central bank conducts a discretionary policy. It would also be of interest to assume a global liquidity trap with inflation inertia. Furthermore, inflation persistence due to an agent's learning or state dependent pricing instead of indexation could cause different outcomes.

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Table 1: Parameter Values

Parameters	Values	Explanation
β	0.9913	Discount Factor
χ	6.25	Elasticity of Output Gap to Real Interest Rate
κ	0.024	Elasticity of Inflation to Output Gap
α	0.66	Price Stickiness
λ_x	0.003	Weight for Output Gap
λ_i	0.077	Weight for Interest Rate
i^*	0.875	Steady State Real Interest Rate
σ_r	0.2445	Standard Deviation of Natural Rate Shock
ρ_r	0.8	Persistence of Natural Rate Shock
σ_μ	0.154	Standard Deviation of Cost-push Shock

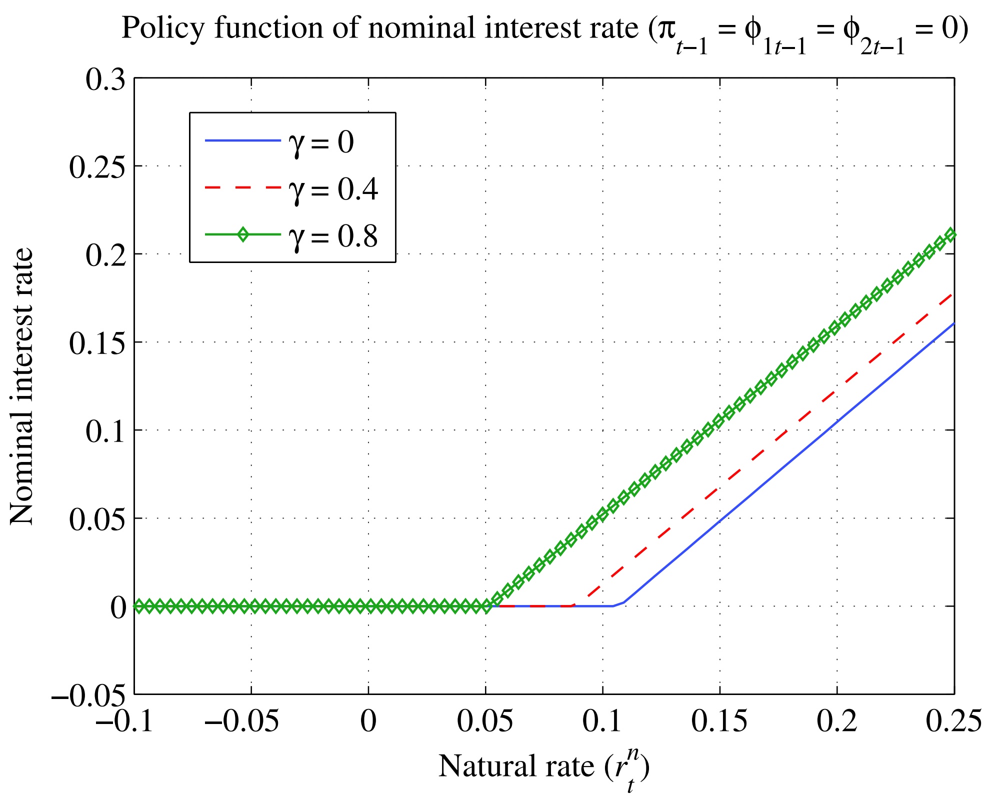

Figure 1: Optimal response of the interest rate against the natural rate shocks for different inflation inertia, where $\pi_{t-1} = \phi_{1t-1} = \phi_{2t-1} = 0$.

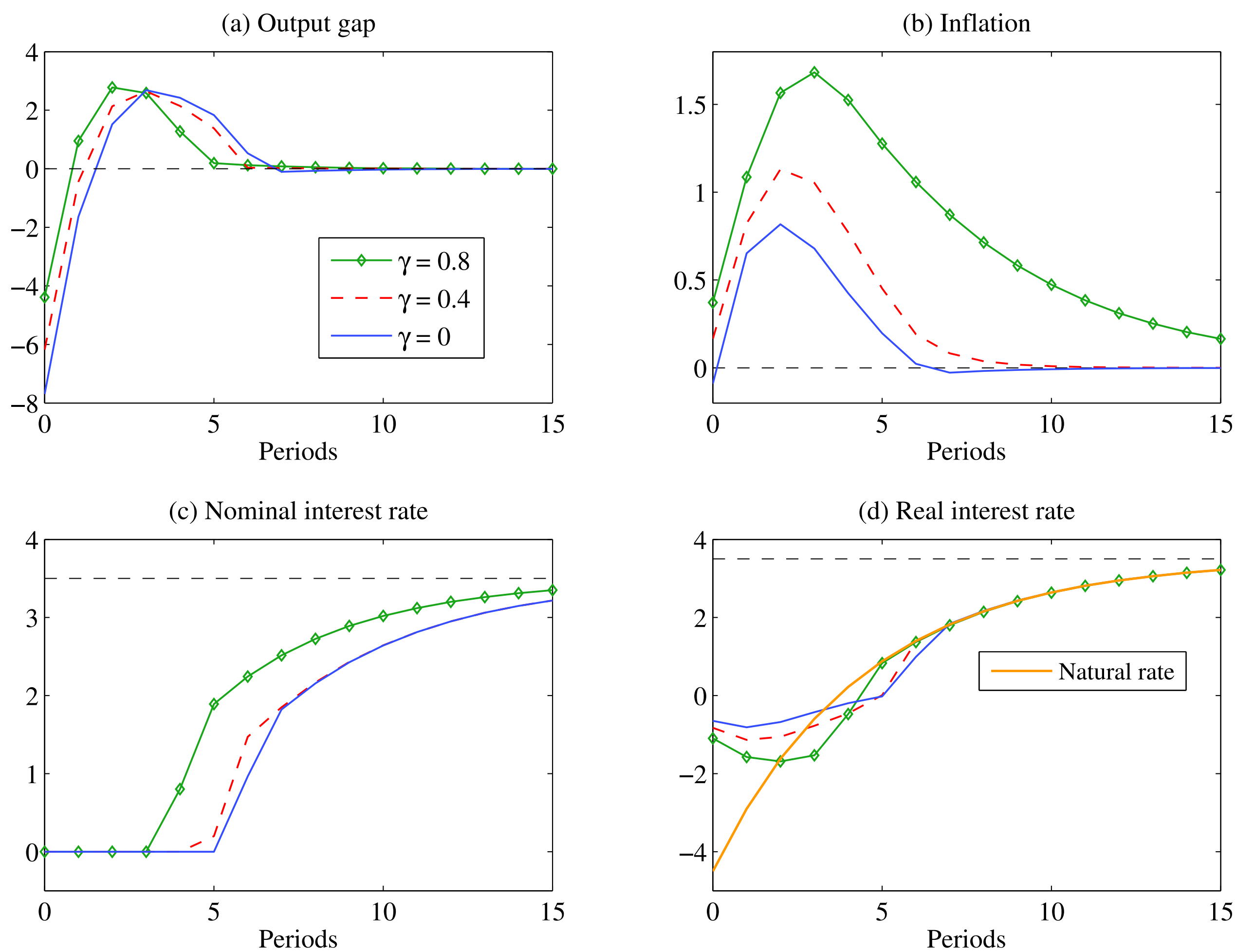

Figure 2: Impulse responses to an annual −8 percent one-time natural rate shock with a persistence 0.8.

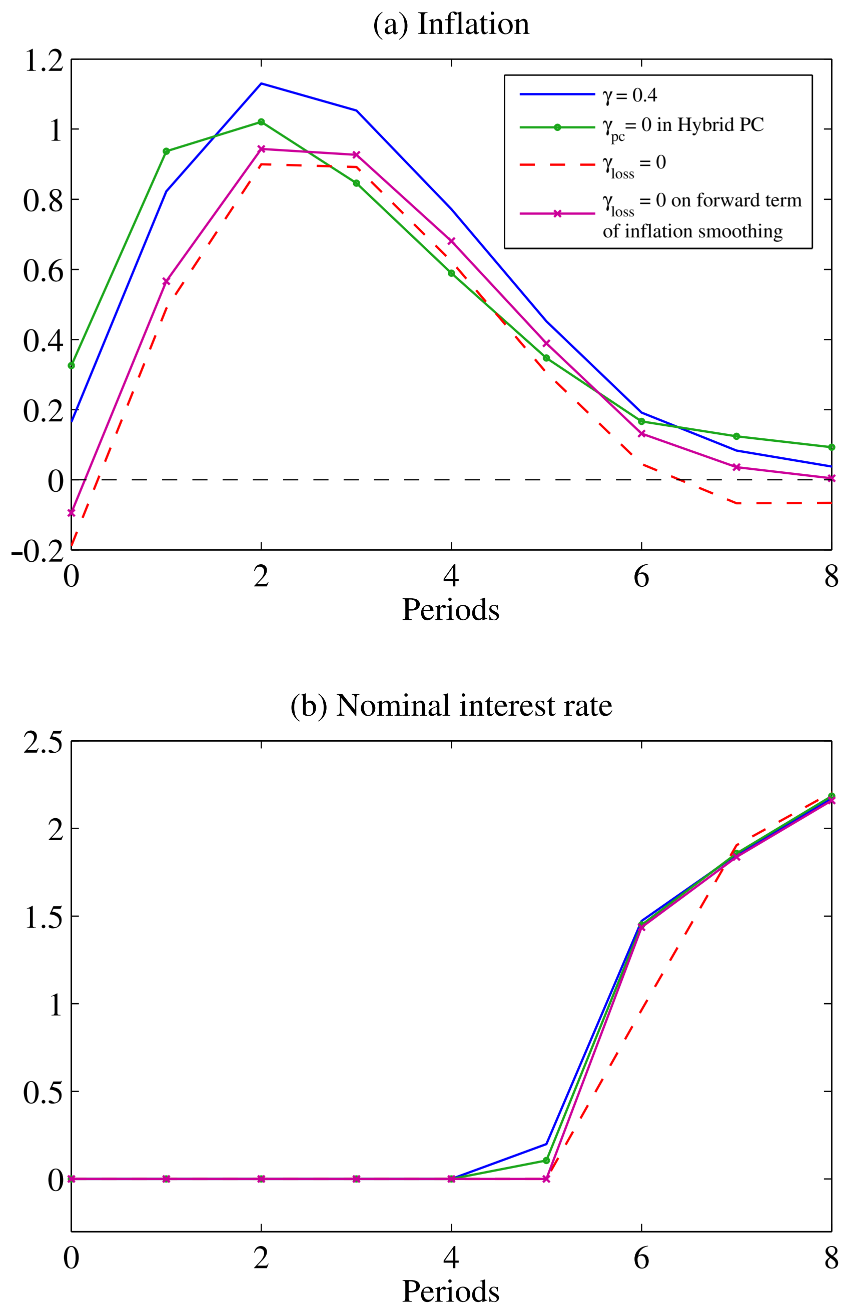

Figure 3: Impulse responses to an annual -8 percent one-time natural rate shock with a persistence of 0.8 when $\gamma = 0.4$. A solid line denotes the case of $\gamma = 0.4$. A dashed line denotes the case of $\gamma_{loss} = 0$. A line marked with circles denotes the case of $\gamma_{pc} = 0$ in a structural equation. A solid line marked with cross mark denotes the case of $\gamma_{loss} = 0$ on a forward term of inflation smoothing.

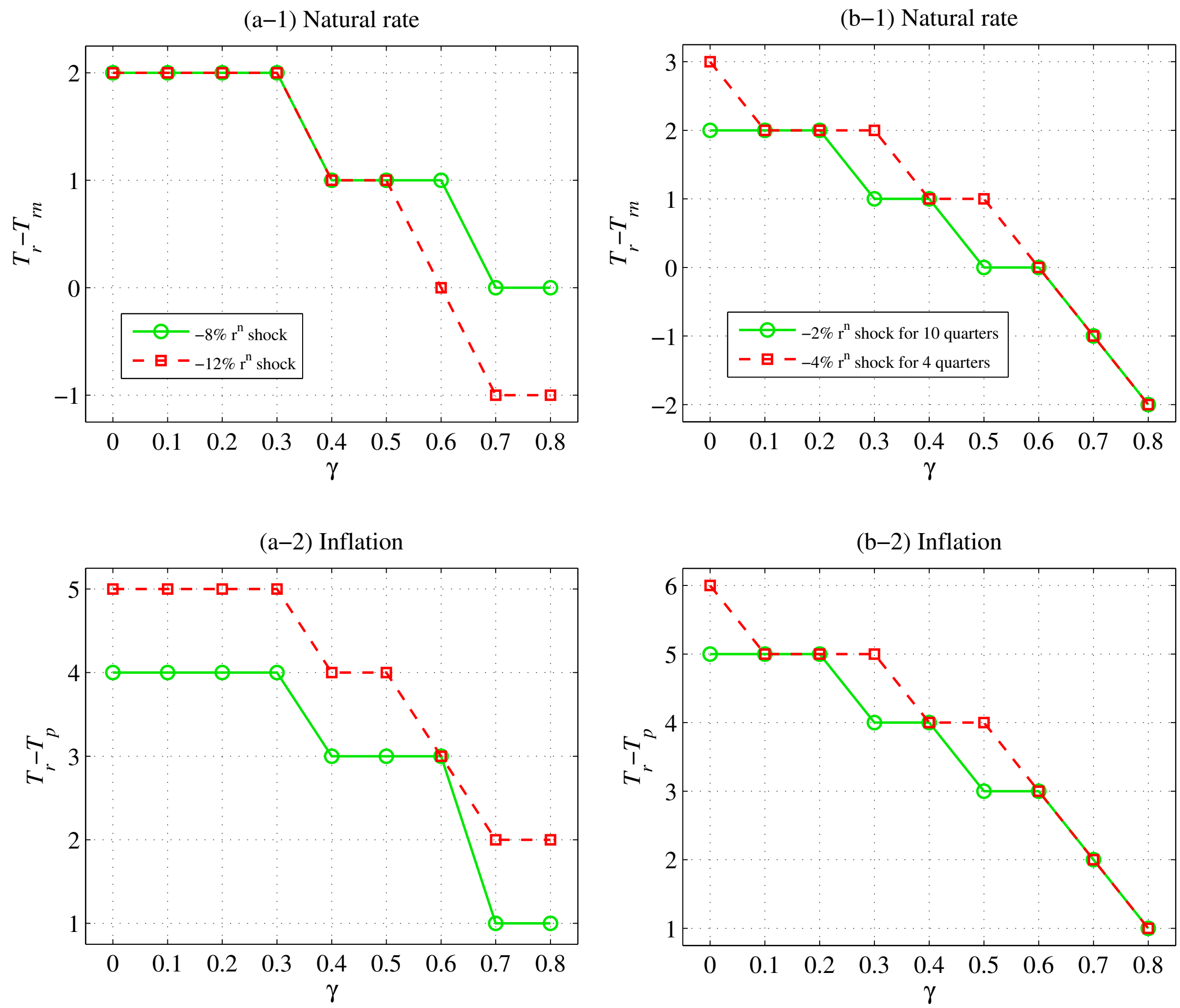

Figure 4: T_r denotes a time when the zero interest rate policy ends, T_{rn} denotes a time when the natural rate returns to zero, and T_p denotes a time when inflation hits its peak. Panels (a-1) and (a-2) denote the cases of annual -8 and -12 percent one-time natural rate shocks with persistence of 0.8. Panels (b-1) and (b-2) denote the cases of annual -2 and -4 percent natural rate shocks for 10 and 4 quarters, respectively.

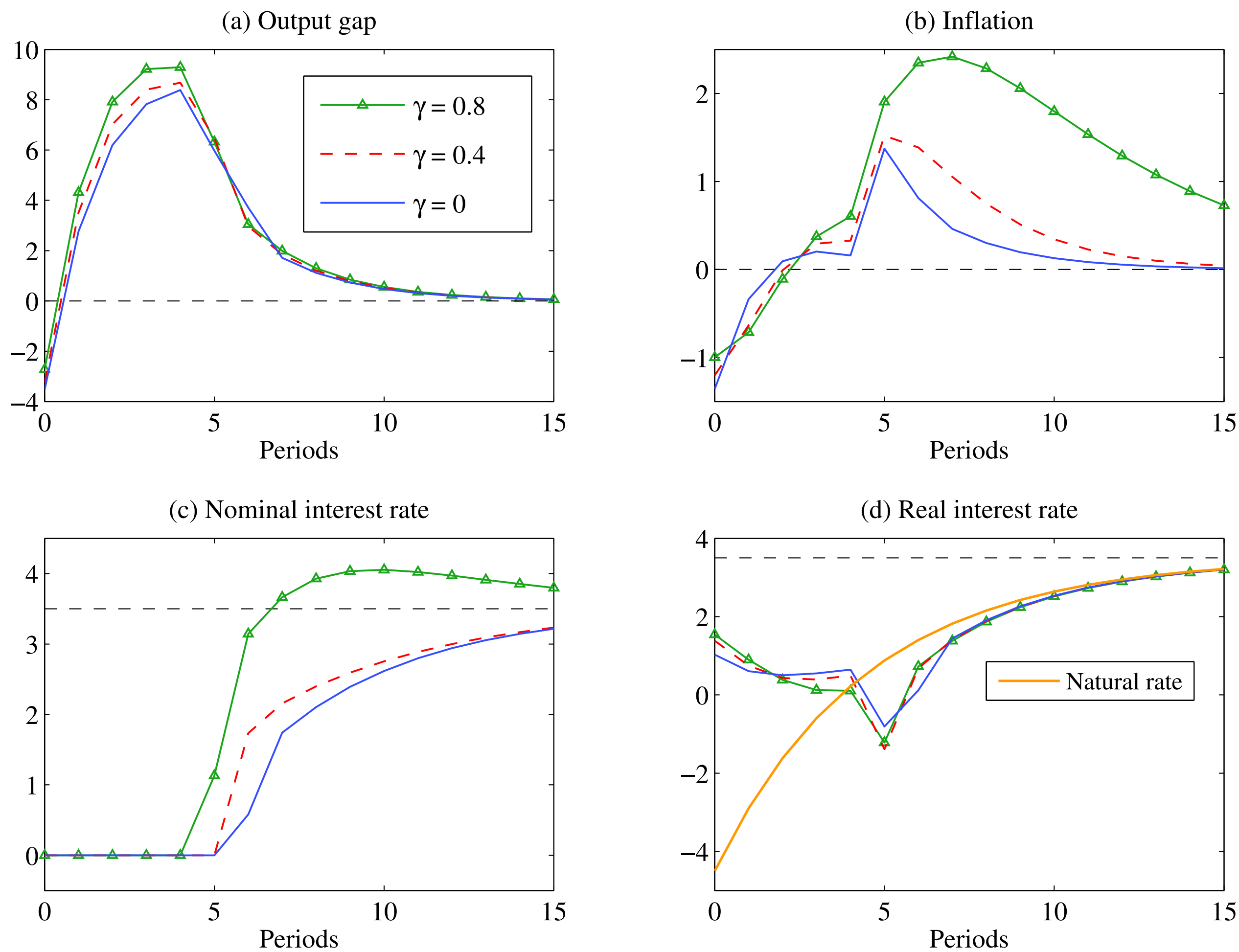

Figure 5: Impulse responses to an annual −8 percent one-time natural rate shock with a persistence of 0.8 and annual −2 percent cost push shocks for 5 quarters.

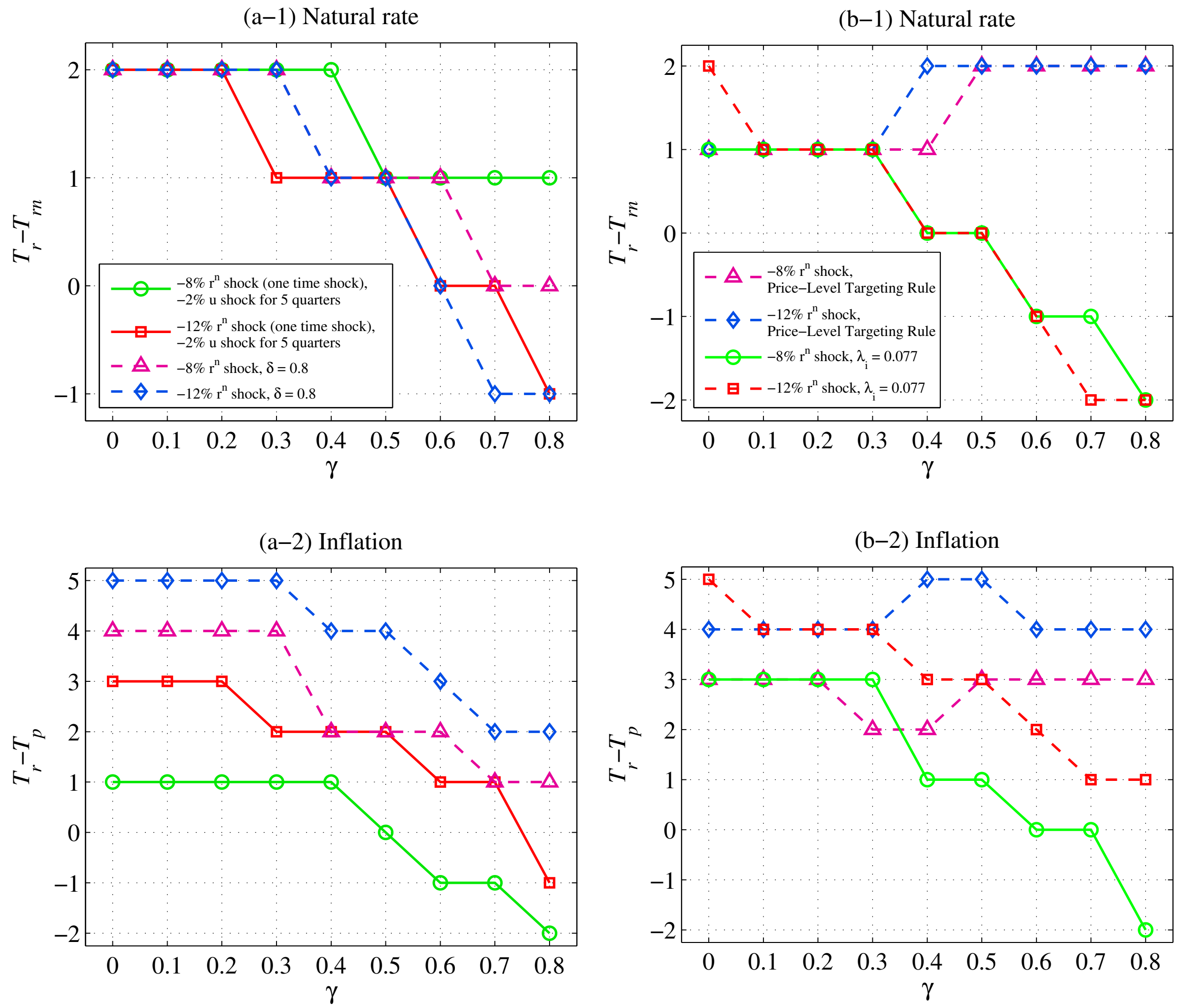

Figure 6: For T_r, T_{rn}, and T_p, see Figure 4. In Panels (a-1) and (a-2), solid lines with circles and squares denote the cases of annual -8 and -12 percent one-time natural rate shocks with persistence of 0.8 accompanied with annual -2 percent cost push shocks for 5 quarters, respectively, and dashed lines with triangles and diamonds denote the cases of annual -8 and -12 percent one-time natural rate shocks with persistence of 0.8, respectively, when $\delta = 0.8$. In Panels (b-1) and (b-2), dashed lines with circles and squares denote the cases of annual -8 and -12 percent one-time natural rate shocks with persistence of 0.8, respectively, when $\lambda_i = 0.077$, and dashed lines with triangles and diamonds denote the cases of annual -8 and -12 percent one-time natural rate shocks with persistence of 0.8, respectively, under a price-level targeting rule.

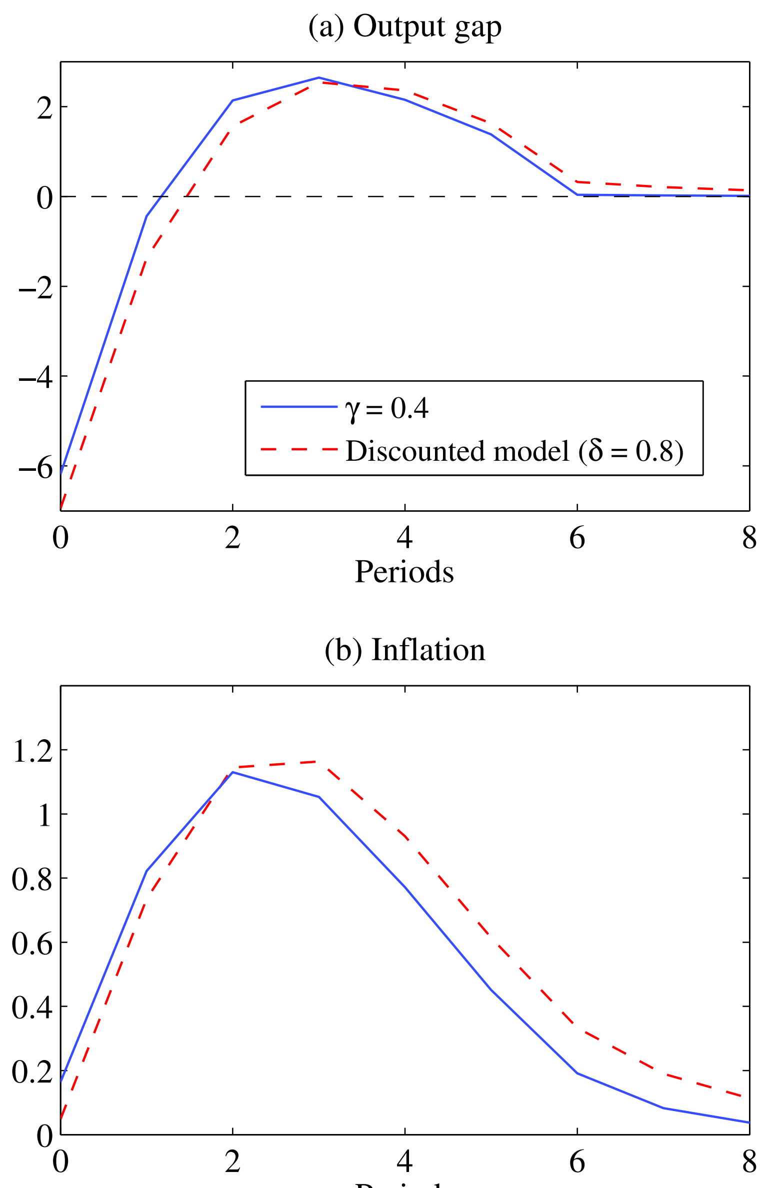

Figure 7: Impulse responses to an annual -8 percent one-time natural rate shock with a persistence of 0.8 when $\gamma = 0.4$. A solid line denotes the case of $\gamma = 0.4$. A dashed line denotes the case of a discounted IS curve with $\delta = 0.8$.

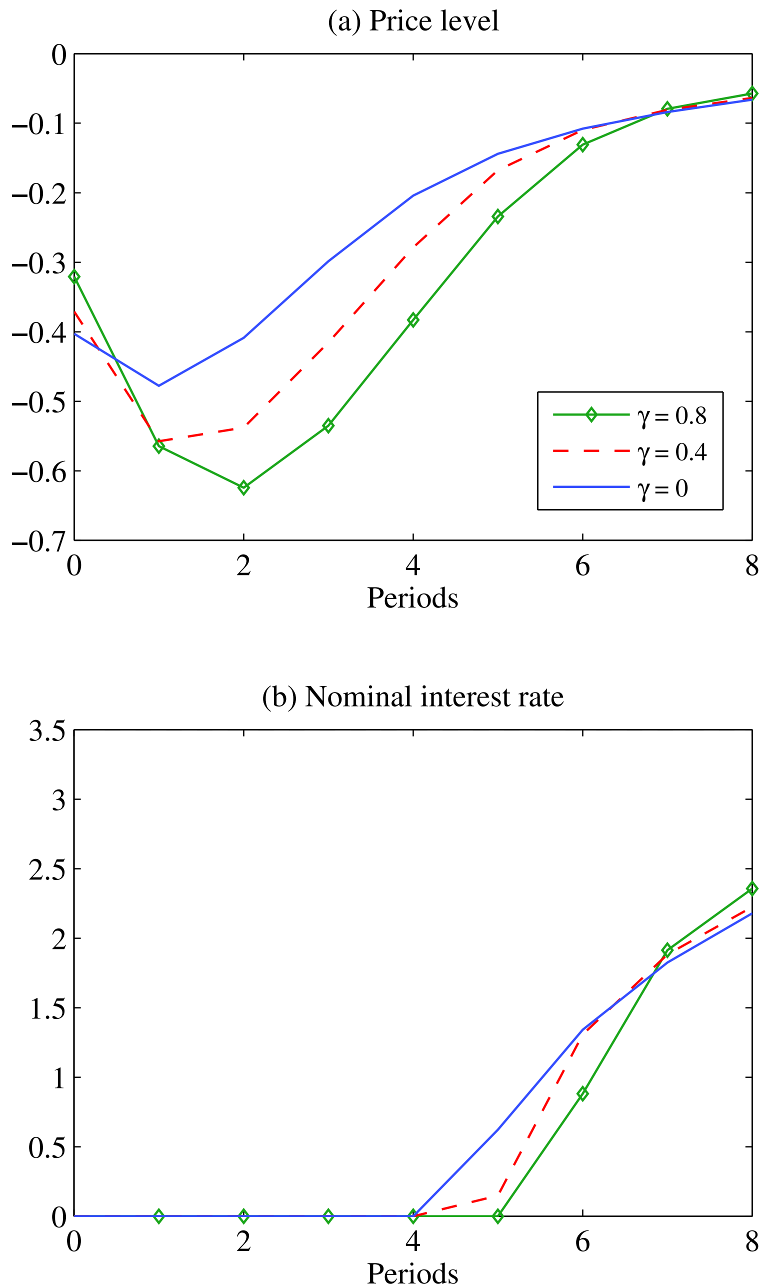

Figure 8: Impulse responses to an annual −8 percent one-time natural rate shock with a persistence of 0.8 under a price level-targeting rule.

Appendix

A Expression in Inflation Rate

We follow the idea of Giannoni and Woodford (2003) to construct equations. We assume $\chi > 0$, $\kappa > 0$, $0 < \beta < 1$, and $0 < \gamma \leq 1$. The equation (11) becomes:[18]

$$\begin{aligned} & \beta\gamma\left(1-\Psi_1 L\right)\left(1-\Psi_2 L\right)\left(1-\Psi_3 L\right)\mathrm{E}_t\phi_{1t+1} \\ = \quad & -\kappa\beta\gamma\left(\mathrm{E}_t\pi_{t+1}-\gamma\pi_t\right)+\kappa\left(\pi_t-\gamma\pi_{t-1}\right)-\beta\lambda_x\gamma\mathrm{E}_t\triangle x_{t+1}+\lambda_x\triangle x_t. \end{aligned}$$

As shown in Giannoni and Woodford (2003), we need one root with $0 < \Psi_1 < 1$ and two roots outside the unit circle to obtain a solution in the model. The two roots are either two real roots $1 < \Psi_2 \leq \Psi_3$ or a complex pair Ψ_2, Ψ_3 of which real parts are greater than one. For any γ, it is the case that

$$\begin{aligned} & -\left(1-\Psi_1 L\right)\left(1-\frac{\Psi_2+\Psi_3}{2}L\right)\phi_{1t} \\ = \quad & \frac{1}{2}\left(\beta\gamma\Psi_3\right)^{-1}\mathrm{E}_t\left[\left(1-\Psi_3^{-1}L^{-1}\right)^{-1}V_t\right]+\frac{1}{2}\left(\beta\gamma\Psi_2\right)^{-1}\mathrm{E}_t\left[\left(1-\Psi_2^{-1}L^{-1}\right)^{-1}V_t\right], \end{aligned}$$

where

$$V_t \equiv -\kappa\beta\gamma\left(\mathrm{E}_t\pi_{t+1}-\gamma\pi_t\right)+\kappa\left(\pi_t-\gamma\pi_{t-1}\right)-\beta\lambda_x\gamma\mathrm{E}_t\triangle x_{t+1}+\lambda_x\triangle x_t.$$

By deconstructing these equations, we have

$$\phi_{1t}-\rho_1\phi_{1t-1}-\rho_2\triangle\phi_{1t-2}=-\frac{1}{2}\left(\beta\gamma\Psi_3\right)^{-1}m_t^I-\frac{1}{2}\left(\beta\gamma\Psi_2\right)^{-1}m_t^{II},$$

where

$$\rho_1=\Psi_1+\frac{\Psi_2+\Psi_3}{2}-\Psi_1\frac{\Psi_2+\Psi_3}{2}>1,$$

$$\rho_2=\Psi_1\frac{\Psi_2+\Psi_3}{2}>0,$$

$$m_t^I=\mathrm{E}_t\left[\left(1-\Psi_3^{-1}L^{-1}\right)^{-1}V_t\right]=\kappa\sum_{i=-1}^{\infty}\alpha_{\pi,i}^I\mathrm{E}_t\pi_{t+i}+\lambda_x\sum_{i=-1}^{\infty}\alpha_{x,i}^I\mathrm{E}_t x_{t+i},$$

[18] Thus, $\beta\gamma=\left(\beta\Psi_1\Psi_2\Psi_3\right)^{-1}$, $\beta\gamma\left(\Psi_1+\Psi_2+\Psi_3\right)=1+\gamma+\beta\gamma$, and $\beta\gamma\Psi_1=1+\gamma-\beta\gamma\left(\Psi_2+\Psi_3-1\right)$.

$$m_t^{II} = \mathrm{E}_t\left[\left(1-\Psi_2^{-1}L^{-1}\right)^{-1}V_t\right] = \kappa\sum_{i=-1}^{\infty}\alpha_{\pi,i}^{II}\mathrm{E}_t\pi_{t+i} + \lambda_x\sum_{i=-1}^{\infty}\alpha_{x,i}^{II}\mathrm{E}_t x_{t+i},$$

$$\alpha_{\pi,-1}^{I} = -\gamma,$$

$$\alpha_{\pi,0}^{I} = 1+\beta\gamma^2-\gamma\Psi_3^{-1},$$

$$\alpha_{\pi,i}^{I} = -\gamma\beta\Psi_3^{-i+1}+\Psi_3^{-i}\left(1+\beta\gamma^2\right)-\Psi_3^{-i-1}\gamma,\ i=1,2,3,...,$$

$$\alpha_{x,-1}^{I} = -1,$$

$$\alpha_{x,0}^{I} = 1+\beta\gamma-\Psi_3^{-1},$$

$$\alpha_{x,i}^{I} = -\gamma\beta\Psi_3^{-i+1}+\Psi_3^{-i}\left(1+\beta\gamma\right)-\Psi_3^{-i-1},\ i=1,2,3,...,$$

$$\alpha_{\pi,-1}^{II} = -\gamma,$$

$$\alpha_{\pi,0}^{II} = 1+\beta\gamma^2-\gamma\Psi_2^{-1},$$

$$\alpha_{\pi,i}^{II} = -\gamma\beta\Psi_2^{-i+1}+\Psi_2^{-i}\left(1+\beta\gamma^2\right)-\Psi_2^{-i-1}\gamma,\ i=1,2,3,...,$$

$$\alpha_{x,-1}^{II} = -1,$$

$$\alpha_{x,0}^{II} = 1+\beta\gamma-\Psi_2^{-1},$$

$$\alpha_{x,i}^{II} = -\gamma\beta\Psi_2^{-i+1}+\Psi_2^{-i}\left(1+\beta\gamma\right)-\Psi_2^{-i-1},\ i=1,2,3,....$$

Finally, we rearrange these equations as:

$$\begin{aligned}
& \phi_{1t}-\rho_1\phi_{1t-1}-\rho_2\triangle\phi_{1t-2} \\
= \quad & \mathrm{E}_t\sum_{i=0}^{\infty}\alpha_{\pi,i}\pi_{t+i}+\mathrm{E}_t\sum_{i=0}^{\infty}\alpha_{x,i}x_{t+i}+\alpha_{\pi,-1}\pi_{t-1}+\alpha_{x,-1}x_{t-1},
\end{aligned}$$

where

$$\alpha_{\pi,-1} = \frac{\kappa}{\beta}\frac{\Psi_2^{-1}+\Psi_3^{-1}}{2},$$

$$\alpha_{x,-1} = \frac{\lambda_x}{\beta\gamma}\frac{\Psi_2^{-1}+\Psi_3^{-1}}{2},$$

$$\alpha_{\pi,i} = -\frac{\kappa}{2\beta\gamma}\left(\Psi_3^{-1}\alpha_{\pi,i}^{I}+\Psi_2^{-1}\alpha_{\pi,i}^{II}\right),$$

$$\alpha_{x,i} = -\frac{\lambda_x}{2\beta\gamma}\left(\Psi_3^{-1}\alpha_{x,i}^{I}+\Psi_2^{-1}\alpha_{x,i}^{II}\right).$$

In particular, for a large γ, coefficients of $\alpha_{\pi,i}$ and $\alpha_{x,i}$ are positives for small i such as -1, 1, 2, and 3. For the parameters in Table 1, when $\gamma = 0.1$ and $\gamma = 0.8$, ($\alpha_{\pi,-1}$, $\alpha_{\pi,0}$, $\alpha_{\pi,1}$, $\alpha_{\pi,2}$, $\alpha_{\pi,3}$) is (0.009, -0.087, -0.041, -0.027, -0.018) and (0.015, -0.023, 0.002, 0.002, 0.002), respectively. When $\gamma = 0.1$ and $\gamma = 0.8$, ($\alpha_{x,-1}$, $\alpha_{x,0}$, $\alpha_{x,1}$, $\alpha_{x,2}$, $\alpha_{x,3}$) is (0.011, -0.006, -0.002, -0.001, -0.001) and (0.002, -0.003, 0.0001, 0.0002, 0.0002), respectively.

B Numerical Algorithm

We solve the central bank's optimization problem by calculating the solution for equations (1) to (3) and equations (6) to (10). Since the zero lower bound (ZLB) introduces nonlinearity in the model, we employ a numerical technique which approximates expected variables.

First of all, we specify the grids for four state variables, r_t^n, ϕ_{1t-1}, ϕ_{2t-1}, and π_{t-1}. Let $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$, and $\mathbf{S}_4$ denote the vector of grids for r_t^n, ϕ_{1t-1}, ϕ_{2t-1}, and π_{t-1}, respectively. A tensor of these grid vectors, defined as $\mathbf{S} \equiv \mathbf{S}_1 \otimes \mathbf{S}_2 \otimes \mathbf{S}_3 \otimes \mathbf{S}_4$, determines the combination of all grids. The size of $\mathbf{S}$ is $N = n_1 \times n_2 \times n_3 \times n_4 = 25000$. As for $\mathbf{S}_1$, we put relatively larger number of grids near the kink point stemming from the ZLB with the aim of mitigating the expected approximation error. The *p.d.f.* for the natural interest rate is discretized by Gaussian Quadrature.

Notice that we can rewrite the complementarity conditions regarding the ZLB, equations (8) to (10), as

$$\min(\max(\chi\phi_{1t}, -i_t), \infty) = 0. \tag{13}$$

In order to employ an algorithmic solution that is designed basically for differentiable functions, we approximate equation (13) by a semismooth function, so called Fischer's equation:

$$\psi^-(\psi^+(\chi\phi_{1t}, -i_t), \infty) = 0,$$

where $\psi^{\pm}(u, v) = u + v \pm \sqrt{u^2 + v^2}$ (c.f., Miranda and Fackler (2004)).

Let $\mathbf{h}_t \equiv (x_t, \pi_t, \phi_{2t})$ denote the vector of forward-looking variables at time t. We need to obtain $\mathbf{h}_t$, i_t, and ϕ_{1t} by solving the central bank's optimization problem, taking state variables as given. In order to calculate the expectations terms, we approximate the time-invariant function for forward-looking variables, $\mathbf{h}$, by a collocation method. Our solution procedure is summarized as follows:

1. Given a particular set of grids for state variables, denoted by $\mathbf{S}^j$, and the initial guess of the functional form for $\mathbf{h}(\mathbf{S}^j)$, denoted by $\mathbf{h}^0(\mathbf{S}^j)$, compute $\mathbf{h}^1(\mathbf{S}^j)$, i_t, and $\phi_{1,t}$ as a solution for equations (1) to (3) and equations (6) to (10). A cubic-spline function is used to interpolate $\mathbf{h}(\mathbf{S^j})$.

2. Repeat step 1 for all $j = 1, \dots N$.

3. Stop if $\|\mathbf{h}^1 - \mathbf{h}^0\|_\infty / \|\mathbf{h}^0\|_\infty < 1.5 \times 10^{-6}$. Otherwise, update the initial functional form as $\mathbf{h}^0 \equiv \mathbf{h}^1$ and go to step 1.

Euler residuals from first order conditions are order of 10^{-3}, which is concentrated mostly around the zero lower bound. Computation time is 8 hours for each γ. The software is Matlab, CPU is Core i7 with 2.90GHz, and Memory is 16GB.

C Proof of Optimal Interest Rate Rule

In this case, we have following first order conditions.

$$-\beta\gamma \mathrm{E}_t \pi_{t+1} + \left(\beta\gamma^2 + 1\right)\pi_t - \gamma\pi_{t-1} - \beta^{-1}\chi\phi_{1t-1} - \beta\gamma \mathrm{E}_t \phi_{2t+1} + (\beta\gamma + 1)\,\phi_{2t} - \phi_{2t-1} = 0, \quad (14)$$

$$\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1} - \kappa\phi_{2t} = 0, \quad (15)$$

$$\lambda_i \left(i_t - i^*\right) + \chi\phi_{1t} - \phi_{3t} = 0, \quad (16)$$

$$i_t \phi_{3t} = 0, \quad (17)$$

$$\phi_{3t} \geq 0, \quad (18)$$

$$i_t \geq 0. \quad (19)$$

where ϕ_1, ϕ_2, and ϕ_3 represent the Lagrange multipliers associated with the IS constraint, the Phillips curve constraint, and the nominal interest rate constraint, respectively.

To prove the optimal interest rate rule, we make use of the Kuhn–Tucker conditions. When the zero lower bound may be binding, we have the following equation from equation (15):

$$\phi_{2t} = \kappa^{-1}(\lambda_x x_t + \phi_{1t} - \beta^{-1}\phi_{1t-1}). \tag{20}$$

By substituting equation (20) into equation (14), we obtain:

$$-\beta\gamma \mathrm{E}_t\phi_{1t+1} + (\beta\gamma+\gamma+1)\phi_{1t} - (1+\gamma+\beta^{-1}(1+\kappa\chi))\phi_{1t-1} + \beta^{-1}\phi_{1t-2}$$
$$= -\kappa(-\beta\gamma \mathrm{E}_t\pi_{t+1} + (\beta\gamma^2+1)\pi_t - \gamma\pi_{t-1}) - \lambda_x(-\beta\gamma \mathrm{E}_t x_{t+1} + (\beta\gamma+1)x_t - x_{t-1}),$$

$$\Rightarrow \mathrm{E}_t\left\{\psi_1(1-\psi_2 L)(1-\psi_3 L)(1-\psi_4 F)\phi^*_{1t}\right\} =$$
$$= \phi^*_\pi(-\beta\gamma \mathrm{E}_t\pi_{t+1} + (\beta\gamma^2+1)\pi_t - \gamma\pi_{t-1}) + \phi^*_x(-\beta\gamma \mathrm{E}_t x_{t+1} + (\beta\gamma+1)x_t - x_{t-1}), \tag{21}$$

where $\phi^*_{1t} = -\chi\lambda_i^{-1}\phi_{1t}$, $\phi^*_\pi \equiv \kappa\chi(\beta\gamma\lambda_i)^{-1}$, $\phi^*_x \equiv \chi\lambda_x(\beta\gamma\lambda_i)^{-1}$, $\psi_1\psi_2\psi_3 = \beta^{-2}\gamma^{-1}$, $\psi_1\psi_2 + \psi_1\psi_3 + \psi_2\psi_3 = (\beta\gamma)^{-1}(1+\beta^{-1}(1+\beta\gamma+\kappa\chi))$, $\psi_1+\psi_2+\psi_3 = (\beta\gamma)^{-1}(1+\beta\gamma+\gamma)$ and $\psi_4 = \psi_1^{-1}$ $(\psi_2 > \psi_3)$. We note that equation (20) is valid with and without the zero lower bound in the system of equations given by equation (14) through equation (19).[19] Giannoni and Woodford (2003) show the optimal interest rate rule without the zero lower bound:

$$\mathrm{E}_t\left\{\psi_1(1-\psi_2 L)(1-\psi_3 L)(1-\psi_4 F)(i_t - i^*)\right\} =$$
$$\phi^*_\pi(-\beta\gamma \mathrm{E}_t\pi_{t+1} + (\beta\gamma^2+1)\pi_t - \gamma\pi_{t-1}) + \phi^*_x(-\beta\gamma \mathrm{E}_t x_{t+1} + (\beta\gamma+1)x_t - x_{t-1}). \tag{22}$$

[19] If the zero lower bound is not binding, from the Kuhn–Tucker conditions equation (16), we can substitute $i_t - i^* = \phi^*_{1t}$ into equation (20), and we obtain the optimal interest rate rule given by equation (22). If the zero lower bound is binding, we can set the optimal interest rates by equation (20) and equation (16). Therefore, equation (20) is valid with and without the zero lower bound.

From equation (20) and equation (22), we obtain:

$$i_t = \phi_{1t}^* + i^*. \tag{23}$$

This relation is true only when the zero lower bound does not bind if i_t cannot take a negative value.[20] In the case where the zero lower bound binds, the Kuhn–Tucker condition equation (16) also holds with $i_t = 0$. Then it must be the case that[21]

$$\phi_{3t} = -\lambda_i(\phi_{1t}^* + i^*).$$

This equation implies that the zero interest rate policy will be terminated when $\phi_{1t}^* + i^*$ becomes positive in equation (21) (or equivalently, the zero interest rate policy will be implemented while $\phi_{1t}^* + i^*$ takes a negative value). From equation (23), we can confirm if i_t could take a negative value in equation (22). Therefore, equation (23) always holds with and without the zero lower bound and i_t becomes positive in equation (22) at the exact same time as that of terminating the zero interest rate policy, which is indicated by ϕ_{1t}^* in equation (21). The above argument can be summarized in the following two equations by redefining $\hat{\imath}_t - i^* = \phi_{1t}^*$, where $\hat{\imath}_t$ can take negative values:

$$i_t = Max(0,\ \hat{\imath}_t),$$

$$\begin{aligned}&\mathrm{E}_t\left\{\psi_1(1-\psi_2 L)(1-\psi_3 L)(1-\psi_4 F)(\hat{\imath}_t - i^*)\right\} = \\ &\qquad \phi_\pi^*(-\beta\gamma\mathrm{E}_t\pi_{t+1} + (\beta\gamma^2+1)\pi_t - \gamma\pi_{t-1}) + \phi_x^*(-\beta\gamma\mathrm{E}_t x_{t+1} + (\beta\gamma+1)x_t - x_{t-1}).\end{aligned}$$

Notice that $\hat{\imath}_t$ can even take a negative value, while i_t cannot under the zero lower bound on nominal interest rates. The above argument completes the proof of the optimal interest rate rule.

[20] This is because ϕ_{1t} takes a negative value, but i_t cannot.

[21] If we substitute $\phi_{3t} = 0$ into equation (16), then we have equation (23) because

$$\lambda_i(i_t - i^*) + \chi\phi_{1t} = 0 \Leftrightarrow\ i_t - i^* = -\lambda_i^{-1}\chi\phi_{1t} = \phi_{1t}^*.$$

D Additional Analyses

In implementing stochastic simulations, it is not easy to secure convergence, which constrains a variety of simulations. In this appendix, we assume deterministic shocks and provide supplemental analyses.

D.1 Low Elasticity of the Output Gap to Real Interest Rates

In analyses of the zero interest rate policy, some papers such as Eggertsson and Woodford (2003a,b), assume a low elasticity of the output gap to real interest rates, which reduces the effectiveness of monetary policy. To check the robustness of the early tightening, we change χ from 6.25 (benchmark) to three alternatives, i.e., $\chi = 3$, $\chi = 1$, and $\chi = 0.5$. Christiano et al. (2005) assume 1 and Eggertsson and Woodford (2003a,b) assume 0.5 for χ.[22] In this simulation, effects of the natural rate shocks on the output gap decrease due to smaller χ in the IS curve. Thus, we adjust size of the natural rate shocks to keep the same size shocks on the output gap.

We assume a case where annual 3 percent negative natural rate shocks with persistence of 0.8 for 10 quarters when $\chi = 3$. As shown in Figure A1, an early tightening becomes more evident as inflation inertia becomes larger. The timing of terminating a zero interest rate policy is earlier by 7 quarters in the case of $\gamma = 0.8$ than that in the case of $\gamma = 0$ in relation to the natural rate of interest. Optimal monetary policy is featured by front-loaded tightening for sufficiently large γ. In the case of $\chi = 1$, we observe similar outcomes to annual 3 percent negative natural rate shocks with persistence of 0.8 for 10 quarters. With a high degree of inflation persistence, a central bank increases its policy rate when the natural rate shock returns to zero, i.e., $T_r - T_{rn} = 0$. Thus, there is no history dependent easing. For $\chi = 0.5$, we assume annual 2 percent negative natural rate shocks with the persistence of 0.8 for 10 quarters since simulation does not converge for larger shocks. Even in this case, an early tightening becomes clearer as

[22] When we assume a different χ, $\lambda_x \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\chi^{-1}+\omega}{1+\omega\theta}$ also changes, where $\theta = 7.88$ is the elasticity of substitution across goods and $\omega = 0.47$ is the elasticity of the desired real wage to the quantity of labor demanded as shown in Woodford (2003).

inflation inertia becomes larger. The timing of terminating a zero interest rate policy is earlier by 6 quarters in the case of $\gamma = 0.8$ than that in the case of $\gamma = 0$ in relation to the natural rate of interest.

D.2 Wage Indexation

As in the case of price persistence, we borrow a model of wage indexation from Woodford (2003). The period loss function, which is derived by approximating the utility function, is given by

$$\mathrm{L}_t = \lambda_p(\pi_t - \gamma_p\pi_{t-1})^2 + \lambda_w(\pi_t^w - \gamma_w\pi_{t-1})^2 + \lambda_x x_t^2, \tag{24}$$

where π_t^w denotes the rate of the change in the nominal wage and λ_w, γ_p, and γ_w are nonnegative parameters satisfying $\lambda_w > 0$, $0 \leq \gamma_p \leq 1$, and $0 \leq \gamma_w \leq 1$, respectively. In addition to the IS curve given by equation (1) and the nonnegativity constraint of the nominal interest rate, the system of the model is given by

$$\pi_t - \gamma_p\pi_{t-1} = \kappa_p x_t + \xi_p(w_t - w_t^n) + \beta(\mathrm{E}_t\pi_{t+1} - \gamma_p\pi_t), \tag{25}$$

$$\pi_t^w - \gamma_w\pi_{t-1} = \kappa_w x_t + \xi_w(w_t^n - w_t) + \beta(\mathrm{E}_t\pi_{t+1}^w - \gamma_w\pi_t), \tag{26}$$

$$w_t = w_{t-1} + \pi_t^w - \pi_t, \tag{27}$$

where w_t and w_t^n denote the real wage and the natural real wage, respectively. λ_p, ξ_p, ξ_w, κ_p, and κ_w are nonnegative parameters satisfying $\lambda_p > 0$, $\xi_p > 0$, $\xi_w > 0$, $\kappa_p > 0$, and $\kappa_w > 0$, respectively.

Intertemporal minimization of equation (24) as in equation (4) subject to equations (1), (10), and (25) – (27) with the nonnegativity constraint on the nominal interest rate yields following first order conditions.

$$\begin{aligned}&\lambda_p(\pi_t - \gamma_p\pi_{t-1}) - \lambda_p\beta\gamma_p(\mathrm{E}_t\pi_{t+1} - \gamma_p\pi_t) - \lambda_w\beta\gamma_w(\mathrm{E}_t\pi_{t+1}^w - \gamma_w\pi_t)\\ &- \frac{\chi}{\beta}\phi_{1t-1} - \phi_{2t-1} + (\gamma_p\beta + 1)\phi_{2t} - \gamma_p\beta\mathrm{E}_t\phi_{2t+1} + \gamma_w\beta\phi_{3t} - \gamma_w\beta\mathrm{E}_t\phi_{3t+1} + \phi_{4t} = 0,\end{aligned}$$

$$\lambda_w(\pi_t^w - \gamma_w\pi_{t-1}) - \phi_{3t-1} + \phi_{3t} - \phi_{4t} = 0,$$

$$\lambda_x x_t - \frac{1}{\beta}\phi_{1t-1} + \phi_{1t} - \kappa_p \phi_{2t} - \kappa_w \phi_{3t} = 0,$$

$$-\xi_p \phi_{2t} + \xi_w \phi_{3t} - \beta \mathrm{E}_t \phi_{4t+1} + \phi_{4t} = 0,$$

$$i_t \phi_{1t} = 0,$$

$$\phi_{1t} \geq 0,$$

$$i_t \geq 0,$$

where ϕ_{1t}, ϕ_{2t}, ϕ_{3t}, and ϕ_{4t} are Lagrange multipliers.

We set $\xi_w = \xi_p = 0.055$, $\kappa_w = \kappa_p = 0.024$, $\lambda_w = \lambda_p = 0.50$, and $\lambda_x = 0.048$ (λ_x is annual) following Giannoni and Woodford (2003) and set $\gamma_p = 0$ to clarify the effect of γ_w. Figure A2 show simulation outcomes. In a case where annual 3 percent negative natural rate shocks with persistence of 0.8 for 10 quarters, the timing of terminating a zero interest rate policy is earlier by 2 quarters in the case of $\gamma = 0.8$ than that in the case of $\gamma = 0$ in response to a shock to the natural rate of interest. A small difference in the timing of terminating a zero interest rate policy is due to two objectives for stabilizing goods inflation, i.e., $\pi_t - \gamma_p \pi_{t-1}$, and wage inflation, i.e., $\pi_t^w - \gamma_w \pi_{t-1}$. To make difference clearer, we need a larger weight for wage inflation compared to goods inflation in the loss function and larger shocks. In a case of $\lambda_w = 5$ and annual 4 percent negative natural rate shocks with persistence of 0.8 for 10 quarters, the timing of terminating a zero interest rate policy is earlier by 6 quarters in the case of $\gamma = 0.8$ than that in the case of $\gamma = 0$ in response to a shock to the natural rate of interest.[23] Moreover, a central bank terminates the zero interest rate policy before the natural rate turns positive, that is, $T_r - T_{rn} = -1$. We observe front-loaded tightening even in the case of wage indexation.

[23] A larger weight for wage inflation is supported by Erceg, C., Henderson, D. and Levin, A. Optimal monetary policy with staggered wage and price contracts. Journal of Monetary Economics, vol. 46(2), pp. 281-313, 2000.

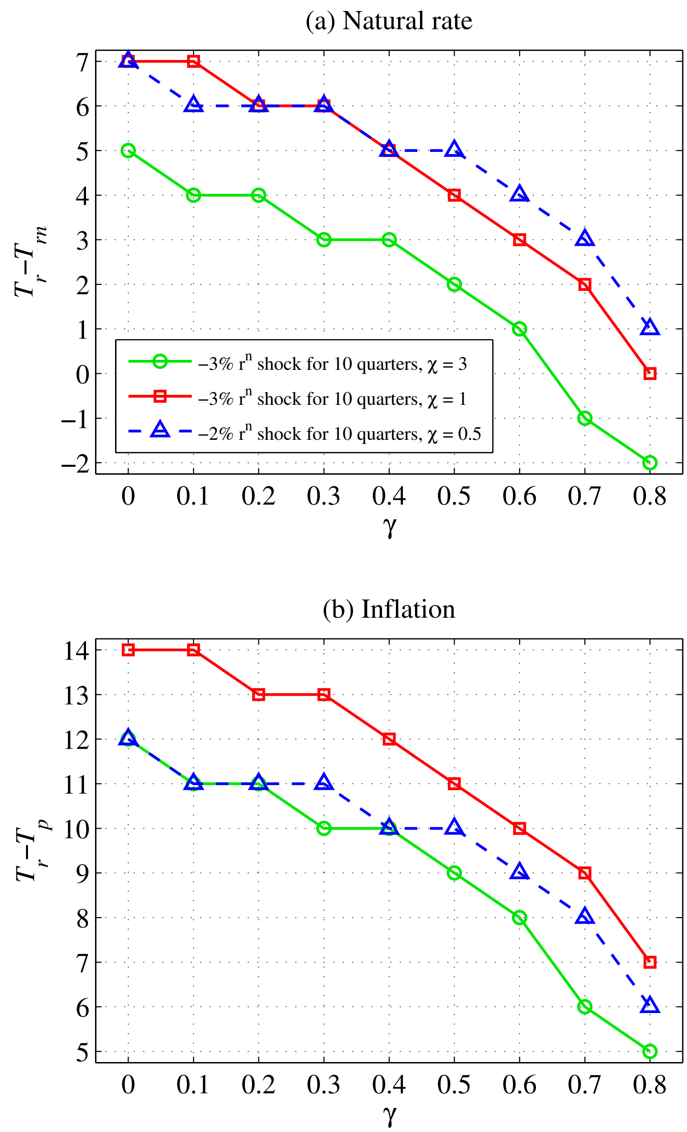

Figure A1: T_r denotes a time when the zero interest rate policy ends, T_{rn} denotes a time when the natural rate returns to zero, and T_p denotes a time when inflation hits its peak. Panels (a) and (b) denote the cases of annual -3 percent natural rate shocks with persistence of 0.8 for 10 quarters when $\chi = 3$ and $\chi = 1$, and denote the case of annual -2 percent natural rate shocks with persistence of 0.8 for 10 quarters when $\chi = 0.5$.

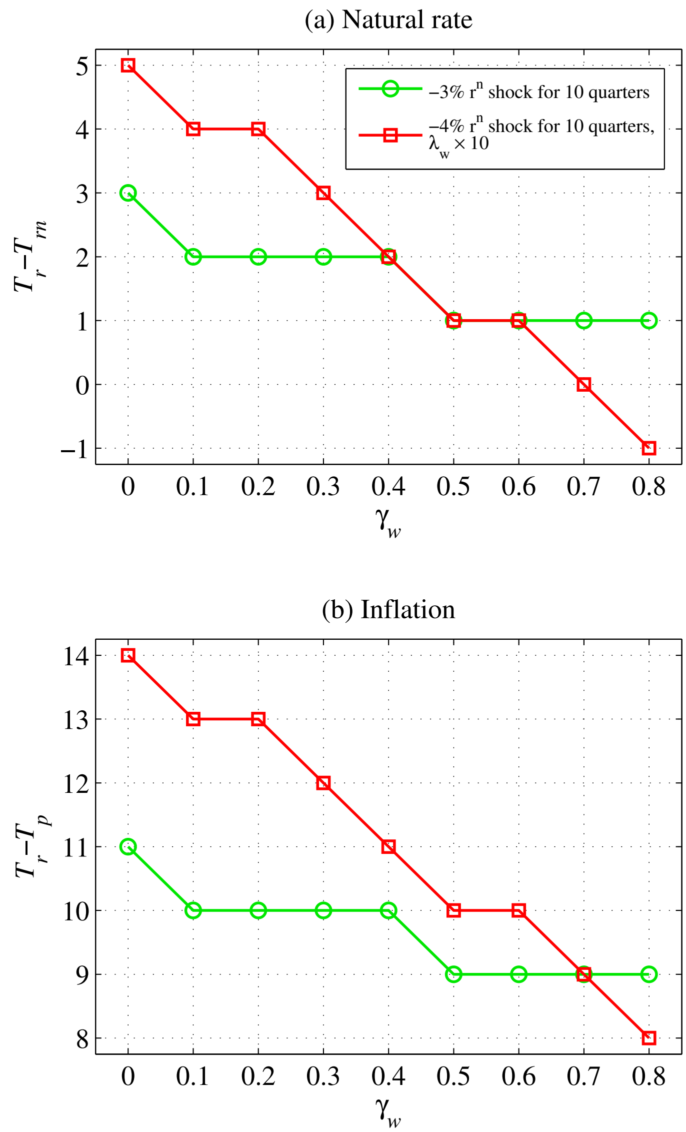

Figure A2: T_r denotes a time when the zero interest rate policy ends, T_{rn} denotes a time when the natural rate returns to zero, and T_p denotes a time when inflation hits its peak. Panels (a) and (b) denote the cases of annual -3 and -4 percent natural rate shocks with persistence of 0.8 for 10 quarters in the model with wage indexation.