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Consumption Taxes and Divisibility of Labor under Incomplete Markets*

Tomoyuki Nakajima† and Shuhei Takahashi‡

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Abstract

We analyze lump-sum transfers financed through consumption taxes in a heterogeneous-agent model with uninsured idiosyncratic wage risk and endogenous labor supply. The model is calibrated to the U.S. economy. We find that consumption inequality and uncertainty decrease with transfers much more substantially under divisible than indivisible labor. Increasing transfers by raising the consumption tax rate from 5% to 35% decreases the consumption Gini by 0.04 under divisible labor, whereas it has almost no effect on the consumption Gini under indivisible labor. The divisibility of labor also affects the relationship among consumption-tax financed transfers, aggregate saving, and the wealth distribution.

Keywords: Transfers; Consumption taxes; Inequality; Uncertainty; Divisibility of labor; Incomplete markets

JEL classification: E62, D31, J22, C68

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1 Introduction

What is the effect of government transfers on inequality and risk sharing when households face labor income uncertainty? Previous studies, such as Flodén (2001) and Alonso-Ortiz and Rogerson (2010), find that increasing lump-sum transfers financed through labor and/or capital income taxes substantially decreases consumption inequality and uncertainty in a general equilibrium model with uninsured earnings risk. However, little is known about the impact of increasing *consumption-tax* financed transfers. Does it help people smooth consumption and does it reduce inequality? What is the impact on efficiency? The present paper analyzes these questions quantitatively.

Although the impact of transfers financed through capital and labor income taxes has been studied, consumption-tax financed transfers could work differently. On one hand, consumption taxes have an advantage in terms of efficiency and revenue. In a representative-agent model, Coleman (2000) finds that increasing consumption taxes and reducing capital and labor income taxes greatly improve efficiency. Further, using a model with infinitely-lived agents and uninsured idiosyncratic wage risk, Feve, Matheron, and Sahuc (2013) find that consumption taxes can finance larger transfers than capital and labor income taxes.[1] On the other hand, consumption taxes seem to have a disadvantage in terms of inequality. For example, Ventura (1999) analyzes a life-cycle model with uninsured idiosyncratic wage risk and finds that a revenue neutral tax reform of replacing (current) capital and labor income taxes with consumption-based taxation widens wealth and income inequality. These findings of previous studies suggest that consumption-tax financed transfers could have implications for efficiency, inequality, and uncertainty that differ from transfers financed through capital and labor income taxes. Given that switching to a tax system that depends more on consumption taxes has been discussed in various countries, the effect of consumption-tax

[1] Feve, Matheron, and Sahuc (2013) find that the Laffer curves for capital and labor income taxes have a peak, while the Laffer curve for consumption taxes does not. These results also hold in a representative-agent model, as shown by Trabandt and Uhlig (2011).

financed transfers is worth investigating.[2]

The model used in the present paper is similar to that used in Aiyagari and McGrattan (1998), Flodén and Linde (2001), Flodén (2001), Pijoan-Mas (2006), Chang and Kim (2006), Alonso-Ortiz and Rogerson (2010), and others, which incorporate endogenous labor supply into an incomplete assets market model analyzed by Aiyagari (1994). There are a large number of infinitely-lived households. Those households differ in their labor productivity and their wage is determined by the marginal product of their labor. Idiosyncratic productivity is stochastic and this generates idiosyncratic wage risk. Asset markets are incomplete and there are only two risk-free assets: government bonds and physical capital. Hence, households cannot fully insure against idiosyncratic wage risk and they partially self-insure through savings and labor supply, leading to a distribution of wealth and labor income across households. In addition, there is a representative firm with the neoclassical production function.

Since labor supply is a quantitatively important channel for consumption smoothing in our setting as shown by Pijoan-Mas (2006) and its nature is potentially important for the impact of transfers, we consider two specifications of labor supply that are widely used in the literature: divisible and indivisible labor. Models with divisible labor and those with indivisible labor both explain cross-sectional distributions of wealth and earnings reasonably well.[3] Nonetheless, to the best of our knowledge, there is no study that compares government policies under the two specifications and previous works analyze transfers either in an economy with divisible labor (Flodén (2001) and Flodén and Linde (2001)) or an economy with indivisible labor (Alonso-Ortiz and Rogerson (2010)). Hence, little is known about how the divisibility of labor changes the impact of transfers on efficiency, inequality, and uncertainty.

We analyze the stationary equilibrium of the above model by raising the consumption tax rate and increasing the amount of transfers endogenously. Specifically, we consider

[2]For the U.S. economy, see, for example, Ventura (1999) and Coleman (2000). Japan raised the consumption tax rate from 5% to 8% in 2014 and plans to raise the tax rate to 10% in 2017. In Japan, there are also ongoing discussions on further increases in consumption taxes. We analyze Japan's economy in Appendix.

[3]For models with divisible labor, see Flodén (2001) and Pijoan-Mas (2006). For models with indivisible labor, see Chang and Kim (2006) and Alonso-Ortiz and Rogerson (2010).

the government that finances lump-sum transfers to households and exogenous government consumption through taxes on consumption, capital income, and labor income as well as government bonds. Previous studies find that capital and labor income taxes as well as bonds affect consumption inequality and uncertainty (Flodén (2001)). Hence, we include them, but fix them in order to focus on consumption taxes and transfers. We calibrate our model to the U.S. economy.

We find that the divisibility of labor is the key to how expanding transfers financed through consumption taxes influences consumption inequality and uncertainty. When labor is divisible, increasing consumption taxes and transfers substantially reduces the consumption Gini.[4] For example, the consumption Gini decreases by about 0.04 from 0.26 to 0.22 as the consumption tax rate rises from the current rate of 5% to 35%. In contrast, when labor is indivisible, the rise in the tax rate brings essentially no change in the consumption Gini. The consumption Gini starts to fall even under indivisible labor as the tax rate is raised further, but the effect is much smaller than that seen under divisible labor.

The intuition behind the above results is as follows. Since leisure is a normal good, households increase leisure with larger transfers. Therefore, aggregate labor hours decrease and the (after-tax) wage rate rises under both divisible and indivisible labor. However, these two cases show different changes in labor hours at the household-level.

As transfers increase under divisible labor, all households reduce their labor hours largely uniformly. Hence, the dispersion of total income (the sum of asset income, labor income, and lump-sum transfers) substantially decreases, leading to large reduction in the consumption Gini. In contrast, under indivisible labor, some households switch employment to nonemployment (i.e., earning zero labor income) with larger transfers, while employed households keep working for the same hours as before under a higher wage. In particular, employment initially decreases mostly among wealth-poor, low-productivity groups, while high-productivity

[4] A similar result holds for the standard deviation of log of consumption and the P50-P10 and P90-P50 ratios of consumption.

households stay employed.[5] Hence, even with a larger amount of lump-sum transfers, the inequality in total income decreases mildly, thereby only slightly reducing the consumption Gini.

The divisibility of labor also affects the relationship between consumption-tax financed transfers and savings, especially those of low-productivity households close to the borrowing limit. As transfers increase under divisible labor, those households reduce labor hours largely uniformly in all states. This implies that they face less volatile future income and consumption, reducing their incentive to save to self-insure. Hence, larger transfers lead to lower aggregate saving and a higher interest rate. Since the number of low-wealth households increases and rich people increase their savings in response to a higher return, wealth inequality widens.

In contrast, as consumption-tax financed transfers increase under indivisible labor, low-productivity, wealth-poor households switch from employment to nonemployment in some states. In other states, they keep working for the same hours as before under a higher wage. Hence, they face more labor income uncertainty. When the level of transfers is low, this increases consumption uncertainty substantially. Hence, those households increase their savings to self-insure. As a result, when the consumption tax rate is raised from the current level of 5%, aggregate saving increases and the interest rate falls initially. The trend continues until the tax rate reaches 35%. Further, since the number of low-wealth households decreases and rich people reduce savings in response to a lower return, wealth inequality initially decreases. As the amount of transfers increases further with a higher consumption tax rate, low-productivity, low-wealth households reduce precautionary savings. Hence, aggregate saving decreases, while the interest rate rises. Wealth inequality increases.

Next, we examine the welfare implication of the above results. To do so, we employ the method of Flodén (2001) and decompose the utilitarian welfare gain into the sum of the gains of changing inequality, uncertainty, and the level. We find that the overall welfare gain

[5] Low-productivity, wealth-rich households always choose not to work.

is largely independent of the divisibility of labor. For example, raising the consumption tax rate from 5% to 35% generates a 1.0% cost of consumption in the economy with divisible labor and a 1.2% cost in the economy with indivisible labor.

However, the compositions of the gains are very different between divisible and indivisible labor. When labor is divisible, the level cost is 4.3%, while the sum of the uncertainty and inequality gains is 3.4%. Hence, increasing consumption-tax financed transfers improves inequality and uncertainty in terms of welfare. In contrast, in the economy with indivisible labor, the level gains by 3.2%, while the sum of the uncertainty and inequality gains is –4.2%. Thus, increasing consumption-tax financed transfers generates a welfare cost on inequality and uncertainty under indivisible labor. Although the further decomposition into the uncertainty and inequality gains varies with assumptions, the sum of the two gains is unchanged. These findings suggest that the divisibility of labor is an important determinant for the effects of increasing consumption taxes and transfers on welfare.

Although the impact of transfers depends on which taxes are used for financing, the mechanism described above works for transfers financed through capital and labor income taxes. Hence, as shown in Appendix, their effects also vary under divisible and indivisible labor. These findings suggest that analyzing lump-sum transfers and other fiscal policies in a model with both intensive and extensive margins, such as the model recently developed by Chang, Kim, Kwon, and Rogerson (2014), would be an important future task.

The remainder of the present paper proceeds as follows. Section 2 presents the model, while Section 3 determines parameter values. Section 4 presents the results and Section 5 conducts robustness checks. Section 6 concludes. In Appendix, we examine transfers financed through capital and labor income taxes for the U.S. economy and also analyze consumption-tax financed transfers for Japan's economy.

2 Model

The model considered here is a neoclassical growth model with uninsured wage risk and endogenous labor supply. We also include the government that finances its consumption and lump-sum transfers to households through taxes and bonds. We explain the model in the order of firms, households, and the government.

2.1 Firms

There is a representative firm. The firm rents capital K and labor N from households, and it produces the good Y. The production function is

$$Y = K^{\theta}(zN)^{1-\theta}, \tag{1}$$

where z is labor-augmenting productivity and $\theta \in (0, 1)$ is the capital share. Productivity grows at a constant rate of g, that is, $z^{'} = (1 + g)z$, where a prime denotes a next-period value hereinafter.

The firm maximizes its static profit taking the rental rate of capital r and the wage rate w as given. The first-order conditions are

$$r = \theta z^{1-\theta} K^{\theta-1} N^{1-\theta} - \delta \tag{2}$$

and

$$w = (1 - \theta) z^{1-\theta} K^{\theta} N^{-\theta}, \tag{3}$$

where $\delta \in (0, 1)$ is the capital depreciation rate. Define $k = K/Y$ and $\tilde{w} = w/Y$. The above conditions are written in a stationary form as

$$r = \frac{\theta}{k} - \delta \tag{4}$$

and

$$\tilde{w} = \frac{(1-\theta)}{N}. \tag{5}$$

2.2 Households

There is a continuum of households (measure one). Households are endowed with one unit of time each period. We consider two cases for labor supply. The first is divisible labor and households have a momentary utility function that shows the constant Frisch elasticity of labor supply and is consistent with balanced growth:

$$\begin{aligned} u(c,h) &= \frac{1}{1-\mu}\left\{c^{1-\mu}[1-\kappa(1-\mu)h^{1+\frac{1}{\varphi}}]^{\mu}-1\right\} \text{ if } \mu \neq 1, \mu > 0 \\ &= \ln c - \kappa h^{1+\frac{1}{\varphi}} \text{ if } \mu = 1, \end{aligned} \tag{6}$$

where c is consumption, $h \in [0,1]$ is labor hours, μ is the coefficient of relative risk aversion, $\kappa > 0$ is the parameter capturing the disutility of labor, and $\varphi > 0$ is the Frisch labor supply elasticity.

The second case assumes indivisible labor and households choose whether to work for $\bar{h}$ hours or not to work at all: $h = \{0, \bar{h}\}$. The utility function in this case is

$$u(c,h) = \ln c - \psi h^{1+\frac{1}{\varphi}}, \tag{7}$$

where $\psi > 0$ is the parameter capturing the disutility of labor.

Households differ in their labor productivity e. These idiosyncratic productivities are mutually independent and follow an AR(1) process: $\ln e' = \rho \ln e + \varepsilon', \varepsilon' \sim N(\mu_\varepsilon, \sigma_\varepsilon^2)$. This is the source of idiosyncratic wage risk.

Households cannot fully insure against wage risk because asset markets are incomplete and there is a borrowing constraint. Specifically, there are only two risk-free assets in the economy. One is physical capital, and the other is government bonds. These two assets are perfect substitutes for households and both assets earn the interest rate r. There are no aggregate shocks and at the beginning of each period, households are distinguished by their current productivity e and total asset holding a. The borrowing constraint requires $a \geq 0$.

To describe the optimization problem of households in a stationary form, define $\tilde{a} = a/Y$. The problem is written as

$$V(\tilde{a}, e) = \max_{\{\tilde{c}, h, \tilde{a}'\}} \left\{ u(\tilde{c}, h) + \beta E[V(\tilde{a}', e')|e] \right\} \tag{8}$$

$$\text{subject to } (1+\tau_c)\tilde{c} + (1+g)\tilde{a}' \leq [1 + (1-\tau_k)r]\tilde{a} + (1-\tau_n)\tilde{w}eh + \chi$$

$$\tilde{c} \geq 0, \tilde{a}' \geq 0$$

$$h \in [0,1] \text{ when labor is divisible}$$

$$h \in \{0, \bar{h}\} \text{ when labor is indivisible,}$$

where $V(\tilde{a}, e)$ is the value function of households, $\tilde{c} = c/Y$, $\beta \in (0,1)$ is the discount factor, and E denotes conditional expectation.[6] The second line is the budget constraint: τ_c is the consumption tax rate, τ_k is the capital income tax rate, τ_n is the labor income tax rate, and $\chi = TR/Y$, where TR is lump-sum transfers from the government to households.

2.3 Government

The government budget constraint is

$$G + TR + rB = B' - B + \tau_n wN + \tau_k r(K+B) + \tau_c C - M, \tag{9}$$

where G is government consumption, B is government debt, C is aggregate consumption, and M is net imports. Dividing (9) by Y leads to the constraint in a stationary form:

[6] For divisible labor, the value function (8) assumes $\mu = 1$, which is our benchmark. When $\mu \neq 1$, the future value needs to be discounted by $\beta(1+g)^{1-\mu}$ instead of β.

	divisible D	indivisible I
β	0.9711	0.9696
g	0.02	
μ	1.0	NA
φ	1.0	
κ	4.80	NA
H	0.25	
$\bar{h}$	NA	0.333
ψ	NA	6.09
γ	0.18	
δ	0.07	
θ	0.38	
τ_c	0.05	
τ_n	0.28	
τ_k	0.36	
m	0.04	
ρ	0.94	
σ_ε	0.205	
μ_ε	–0.01162	
b	0.63	

Table 1: Baseline parameter values.

$$\gamma + \chi + rb = (1+g)b^{'} - b + \tau_n \tilde{w} N + \tau_k r(k+b) + \tau_c c_{agg} - m, \tag{10}$$

where $b = B/Y$, $c_{agg} = C/Y$, and $m = M/Y$. In the stationary equilibrium, $b = b^{'}$.

3 Benchmark Parameter Values

We calibrate the above model to the U.S. economy. Table 1 lists the benchmark parameter values. One period corresponds to one year.

The parameter values are mostly taken from Trabandt and Uhlig (2011). The growth rate of real GDP g is 2.0% per year. The capital depreciation rate δ is 0.07 and the capital share θ is 0.38. The share of government consumption in GDP γ is 0.18. The capital income tax rate τ_k is 36%, while the labor income tax rate τ_n is 28%. The consumption tax rate τ_c

is 5%. The net import m is 4% of GDP. The government debt b is 63% of GDP.

Next, we parameterize the utility function. For the economy with divisible labor, we set $\varphi = 1.0$ and adjust κ so that the total labor hours is $H = 0.25$, following Trabandt and Uhlig (2011). We set $\mu = 1.0$ to maintain the comparability with the case with indivisible labor. For the economy with indivisible labor, we set $\bar{h} = 0.333$, which is the standard value used in the literature. We then choose the disutility parameter ψ so that total labor hours H is 0.25.

The stochastic process for idiosyncratic productivity e is taken from Alonso-Ortiz and Rogerson (2010) and we assume $\rho = 0.94$ and $\sigma = 0.205$. As Alonso-Ortiz and Rogerson (2010) argue, these values are in line with conventional estimates. The AR(1) process is approximated with a 17-state Markov chain using Tauchen (1986)'s method. We adjust μ_ε so that the mean of e is 1.0.

Lastly, we choose the discount factor β so that the after-tax rate of return on savings $(1 - \tau_k)r$ is 4.0% at the benchmark parameter values, following Trabandt and Uhlig (2011). The result is $\beta = 0.9711$ for the economy with divisible labor and $\beta = 0.9696$ for the economy with indivisible labor.

4 Results

We change the consumption tax rate τ_c from 0% to 90% by 5% each and adjust government transfers endogenously in the stationary equilibrium. As shown in Figure 1, the amount of transfers TR and its level relative to output χ increase monotonically in both cases of divisible and indivisible labor.[7] In contrast, we fix government consumption, government bonds, and net imports relative to output (γ, b, m) at the benchmark parameter values, following Aiyagari and McGrattan (1998) and Flodén (2001).[8]

[7] There is no peak in the consumption Laffer curve for the case with divisible labor, as found by Feve, Matheron, and Sahuc (2013). We find that the same is true for the case with indivisible labor.

[8] Aiyagari and McGrattan (1998) and Flodén (2001) did not include net imports. The main results of the present paper do not change when we fix the amounts of the government consumption, government bonds,

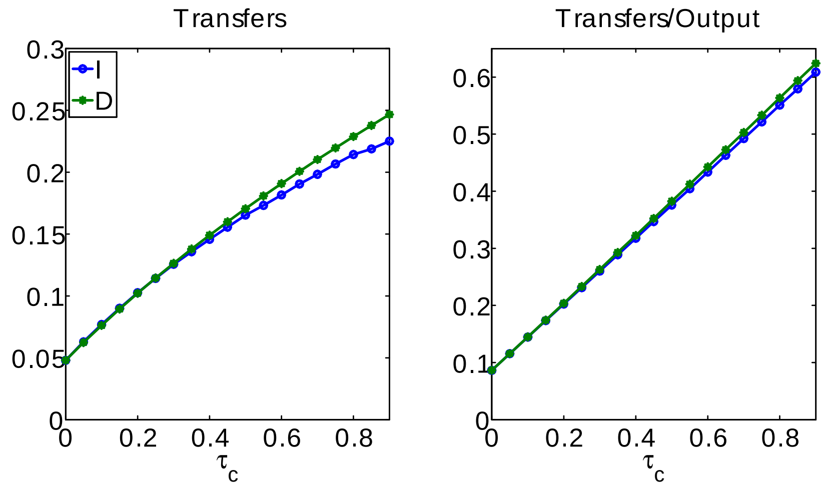

Figure 1: Transfers. I: baseline indivisible labor. D: baseline divisible labor.

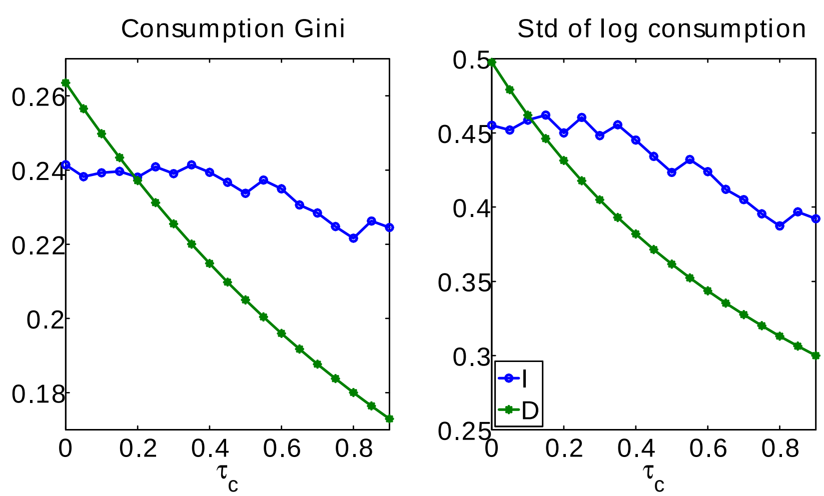

Figure 2: Consumption inequality and uncertainty. I: baseline indivisible labor. D: baseline divisible labor.

Figure 2 shows how two measures of consumption inequality and uncertainty, the Gini coefficient for consumption and the standard deviation of log of consumption, vary with the consumption tax rate.[9] When labor is divisible, increasing transfers financed through consumption taxes reduces the two measures substantially. For example, expanding transfers by increasing the tax rate from the current level of 5% to 35% reduces the consumption Gini by 0.04 from 0.26 to 0.22. The standard deviation of log of consumption decreases by 0.09 from 0.48 to 0.39.

In contrast, increasing the consumption tax rate and expanding transfers reduce consumption inequality and uncertainty only mildly in the case of indivisible labor. When the tax rate is raised from 5% to 35%, both the consumption Gini and the standard deviation of log of consumption are essentially unchanged. Although both measures start to decrease when the tax rate is raised further, the effect is much smaller compared to that seen under divisible labor.

What explains the difference in the movement of consumption inequality between divisible and indivisible labor? Since leisure is a normal good, households increase leisure with a larger amount of transfers. Therefore, as shown in Figure 3, labor input, measured in both row and efficiency-weighted hours, decreases and the after-tax wage rate rises under divisible and indivisible labor. However, micro-level changes in hours are different.

When labor is divisible, all households reduce labor hours relatively uniformly in the face of larger transfers. Hence, with larger lump-sum transfers, the cross-sectional dispersion of total income (the sum of asset income, labor income, and transfers) shrinks, as evidenced by the movement of the Gini coefficient of total income in Figure 3.[10] Hence, consumption inequality substantially decreases. In contrast, when transfers increase under indivisible labor, employment decreases mostly among small-wealth, low-productivity households. Most

and net imports to those under the initial tax rate ($\tau_c = 0.05$ in our case), as in Trabandt and Uhlig (2011). See the next section for the detail.

[9] The P50-P10 and P90-P50 ratios of consumption show a similar pattern. Under divisible labor, both ratios monotonically decrease with the consumption tax rate. In contrast, under indivisible labor, a hike in the consumption tax rate initially increases and then decreases the ratios.

[10] The standard deviation of log of total income exhibits a similar pattern.

of high-productivity, wealth-rich households keep working for the same hours as before under a higher wage, whereas low-productivity, wealth-rich households stay in nonemployment. Accordingly, the inequality in labor income increases substantially. Hence, even though lump-sum transfers increase with a higher consumption tax rate, the inequality in total income decreases only slightly, reducing consumption inequality only mildly.[11]

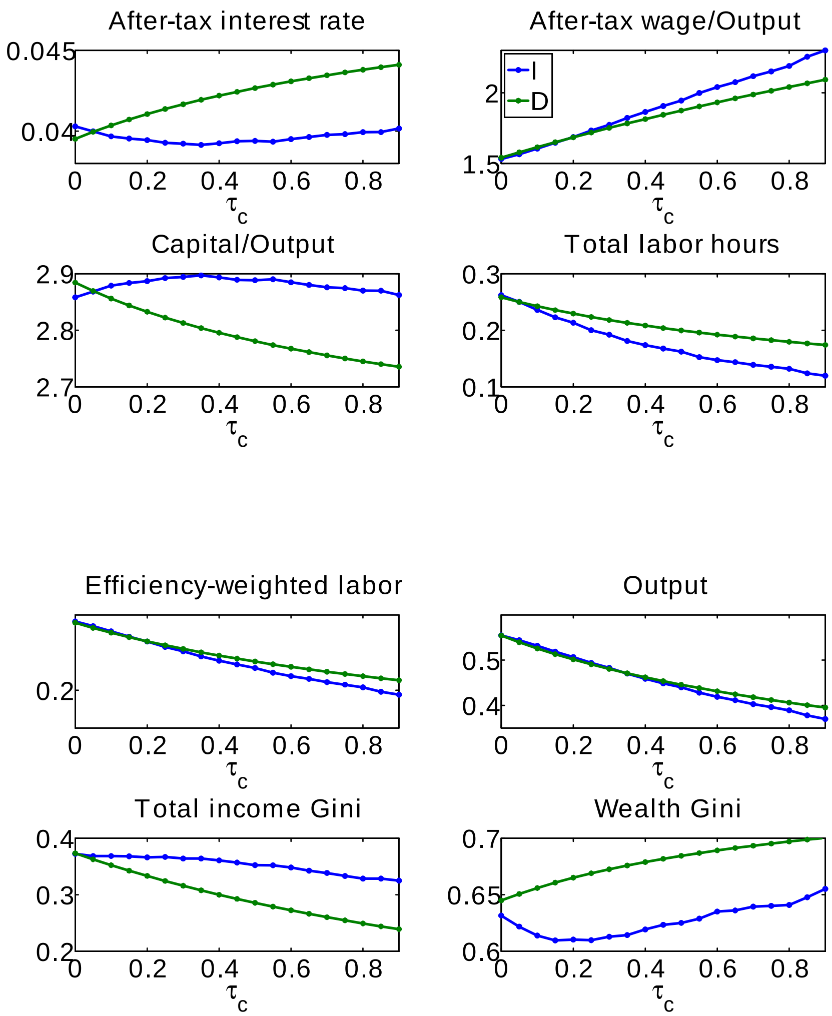

Figure 3: Aggregate variables. I: baseline indivisible labor. D: baseline divisible labor.

The divisibility of labor also affects the saving behavior, especially that of low-wealth households close to the borrowing limit. When transfers increase under divisible labor, those

[11] These adjustments in household-level hours can be inferred from the movements of row and efficiency-weighted hours. Efficiency-weighted labor decreases much less substantially than total labor hours under indivisible labor. The difference is smaller under divisible labor.

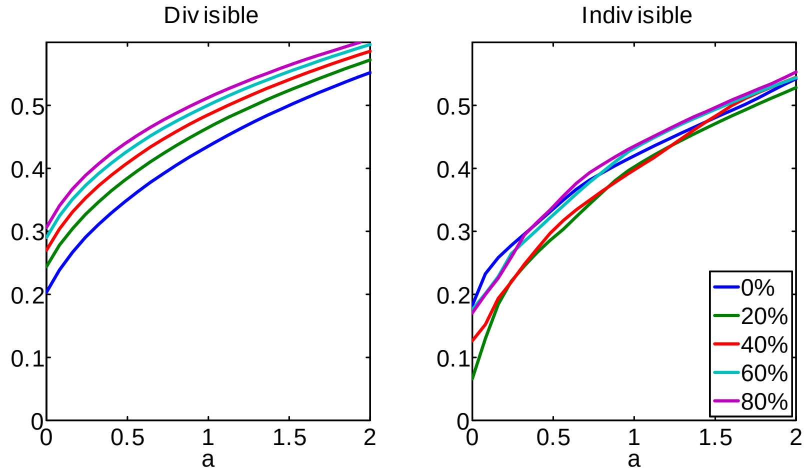

Figure 4: Cumulative asset distribution. We plot the results at the consumption tax rates of 0%, 20%, 40%, 60%, and 80%.

households reduce their labor hours largely uniformly in all states. Accordingly, they face less volatile total income and consumption, reducing their incentive to save to self-insure. This is evidenced by the result in Figure 4 that the number of low-wealth households increases with the consumption tax rate. Hence, aggregate saving decreases as consumption-tax financed transfers increase, which leads to a higher return on savings and a lower capital-output ratio. The wealth Gini monotonically increases with the consumption tax rate because the number of low-wealth households increases and the higher return also increases the savings of wealth-rich households.

In contrast, when transfers increase under indivisible labor, low-productivity, low-wealth households switch from employment to nonemployment in some states, increasing the probability of zero labor income. In other states, those households work for the same hours as before under a higher wage rate. This leads to greater labor earnings uncertainty. When the level of transfers is relatively low, this also increases consumption uncertainty substantially, leading low-productivity, low-wealth households to increase their savings to self-insure. As a result, the number of low-wealth households initially decreases as the consumption tax rate rises, as shown in Figure 4. Accordingly, in the economy with indivisible labor, the capital-output ratio (aggregate saving) increases and the interest rate falls as the tax rate rises from the current level of 5% to 35%. In other words, if we measure the degree of precautionary savings by the interest rate differential from the rate in the complete market counterpart, which is independent of the consumption tax rate, then precautionary savings increase as public transfers increase in the economy with indivisible labor. Further, since the number of low-wealth households decreases and wealth-rich households decrease their savings in response to a lower interest rate, the wealth Gini is initially decreasing in the consumption tax rate, in contrast to the case with divisible labor.

Next, we explore the welfare implication of increasing transfers financed through consumption taxes. For this purpose, we follow the method of Flodén (2001) and decompose the gain in the utilitarian welfare into the welfare gains arising from changes in the level (efficiency), uncertainty, and inequality. First, the utilitarian welfare measure is given by

$$U = \frac{\ln Y_0}{1-\beta} + \frac{\beta \ln(1+g)}{(1-\beta)^2} + \int\int V(\tilde{a}, e) d\Gamma(\tilde{a}, e), \tag{11}$$

where Y_0 is the output that realizes at the initial level of productivity ($z_0 = 1.0$ for normalization) and $\Gamma(\tilde{a}, e)$ is the stationary distribution of households over assets and idiosyncratic productivity.

Consider a change in the consumption tax rate from the initial rate of $\tau_c^1 = 0.05$ to τ_c^2. Let ω_U be the utilitarian welfare gain of the policy change. The welfare gain is expressed as a percent of the consumption that is enjoyed at the initial tax rate.[12] Then, as shown by Flodén (2001), ω_U is expressed as follows:

$$\omega_U = (1 + \omega_{lev})(1 + \omega_{unc})(1 + \omega_{ine}) - 1, \tag{12}$$

where ω_{lev} is the level gain, ω_{unc} is the uncertainty gain, and ω_{ine} is the inequality gain. The level gain ω_{lev} captures the welfare gain from a change in the level of aggregate consumption

[12] More specifically, the number shows how much consumption at the initial tax rate must increase at all states and dates in order to achieve the utilitarian welfare attained at the new tax rate.

(a change in aggregate leisure is compensated), assuming no uncertainty and no inequality (a representative household). The uncertainty gain ω_{unc} is the gain in the uncertainty cost, which is computed by the welfare difference between having the average consumption/leisure and having the average certainty-equivalent consumption/leisure. Lastly, the inequality gain ω_{ine} is the difference in the cost of inequality, which is computed by the difference between the welfare of having the average certainty-equivalent consumption/leisure and the average welfare of having certainty-equivalent consumption/leisure.

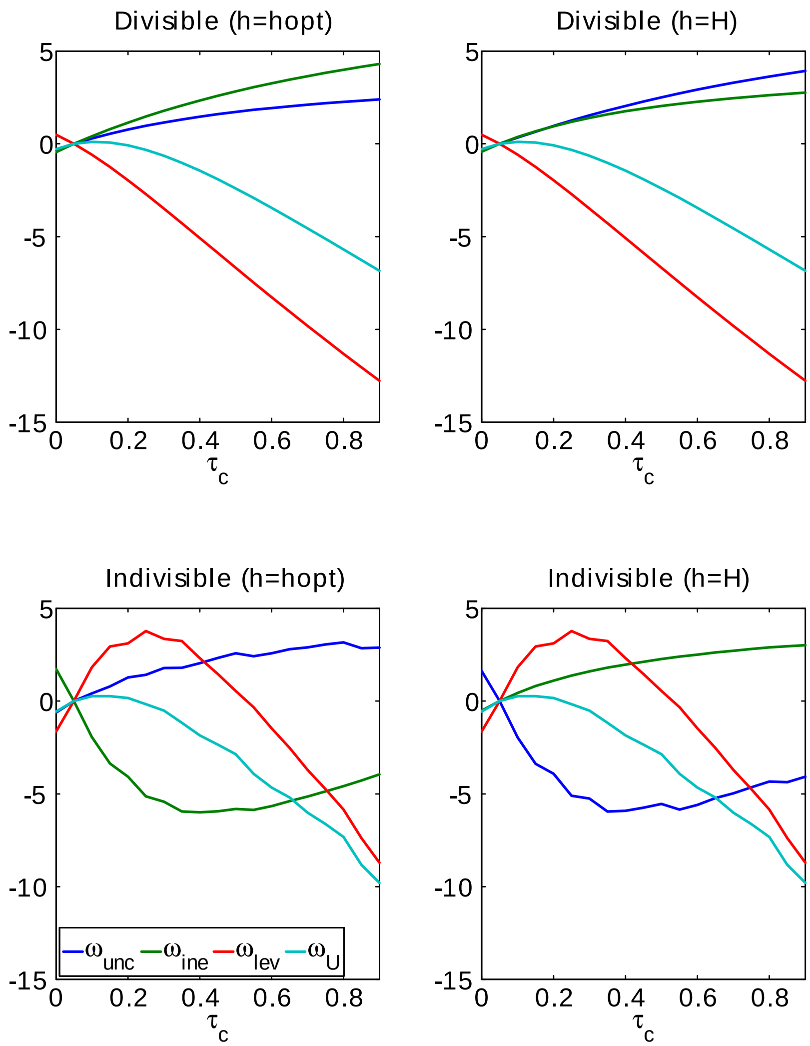

Figure 5: Welfare. The horizontal axis shows the welfare gain, which is expressed as a percent of consumption at the initial tax rate ($\tau_c = 0.05$). We determine the certainty-equivalent leisure in two ways: setting hours to the current level ($h = hopt$) and to the economy average ($h = H$).

The result is shown in Figure 5. As a pair of certainty-equivalent consumption and leisure is not unique, following Flodén (2001), we fix the certainty-equivalent leisure in two ways: setting labor hours to the optimal choice in the current period ($h = hopt$) and the economy average ($h = H$). As shown below, this only affects the decomposition between the inequality and uncertainty gains. The total welfare gain of the policy change, ω_U, is largely independent of the divisibility of labor. The optimal consumption tax rate, where ω_U is the highest, is 10% for both cases. Further, when raising the tax rate from 5% to 35%, the overall welfare cost is 1.0% for the case with divisible labor, while the cost is 1.2% for the case with indivisible labor.

However, substantial differences are observed in the decompositions of the level, uncertainty, and inequality gains. In the economy with divisible labor, increasing consumption taxes and transfers worsens the level, but it improves uncertainty and inequality. When the tax rate is raised from 5% to 35%, for example, the level costs by 4.3%, while the sum of the inequality and uncertainty gains is 3.4%. The further decomposition into the inequality and uncertainty gains depends on how we determine the certainty-equivalent leisure. When $h = hopt$, the inequality gain is 2.1% and the uncertainty gain is 1.3%. When $h = H$, the inequality gain is 1.6% and the uncertainty gain is 1.8%. Nonetheless, the sum of the two is unchanged.

In contrast, in the case with indivisible labor, increasing consumption taxes and transfers increases the sum of the costs of inequality and uncertainty, while it improves the level, at least initially. For example, when the consumption tax rate is raised from 5% to 35%, the level gain is 3.2%, while the sum of the uncertainty and inequality gains is –4.2%. The positive level gain arises from the following reason. As consumption taxes and transfers increase, less productive households stop working, and hence the average productivity of employed households increases.[13] Accordingly, while aggregate labor hours decrease and aggregate leisure increases substantially, aggregate consumption decreases only slightly, increasing the

[13] Alonso-Ortiz and Rogerson (2010) show that a similar result holds when labor income taxes increase.

level gain. As in the case of divisible labor, the decomposition into the inequality and uncertainty gains depends on how we determine the certainty-equivalent leisure. In particular, the signs of the gains differ between $h = hopt$ and $h = H$. However, the sum of the two is unchanged. It is still negative even when the consumption tax rate reaches 90%. The sum is only positive when the tax rate is reduced.

To summarize, increasing consumption taxes and expanding transfers substantially reduce the measures of consumption inequality and uncertainty in the economy with divisible labor, while not very much in the economy with indivisible labor. The welfare decomposition implies a similar result. When labor is divisible, increasing consumption-tax financed transfers generates welfare gains in terms of inequality and uncertainty. In contrast, when labor is indivisible, such a policy change increases the welfare costs of inequality and uncertainty.

5 Robustness Checks

In this section, we show the robustness of the results in the previous section. First, we change the relative risk aversion μ and the Frisch labor supply elasticity φ for the economy with divisible labor. Since most studies assume the relative risk aversion between one and two, we consider $\mu = 2.0$. We also examine a low Frisch elasticity ($\varphi = 0.5$), following Flodén (2001).[14] Second, for the case with indivisible labor, we consider larger labor hours $H = 0.33$ and $\bar{h} = 0.407$, following Alonso-Ortiz and Rogerson (2010). Third, we set the process of idiosyncratic productivity to that used in Aiyagari and McGrattan (1998): $\rho = 0.60$ and $\sigma = 0.24$. The persistence is close to the lower bound of the persistence assumed in previous works. The resulting cross-sectional dispersions of wages and wealth are substantially smaller than those in the benchmark case and those seen in the U.S. In all the cases, we reset the discount factor β and the disutility parameter (κ or ψ) so that the after-tax interest rate $(1 - \tau_k)r$ is 4% and aggregate labor hours hit the target ($H = 0.25$

[14] The welfare measure in (11) is modified in the case of $\mu = 2.0$. We also examined the case with a high elasticity ($\varphi = 2.0$), but the main results of the present paper did not change.

	divisible high risk aversion D-RA	divisible low Frisch elasticity D-FL	indivisible large labor I-H	divisible less persistent risk D-IP	indivisible less persistent risk I-IP
β	0.9814	0.9729	0.9714	0.9794	0.9791
μ	2.0	1.0	NA	NA	NA
φ	1.0	0.5	1.0	1.0	1.0
κ	2.84	13.05	NA	5.22	NA
H	0.25	0.25	0.33	0.25	0.25
$\bar{h}$	NA	NA	0.407	NA	0.333
ψ	NA	NA	4.26	NA	6.18
ρ	0.94	0.94	0.94	0.60	0.60
σ_ε	0.205	0.205	0.205	0.24	0.24
μ_ε	–0.01162	–0.01162	–0.01162	–0.0182	–0.0182

Table 2: Baseline parameter values.

or 0.33). Table 2 summarizes those parameter values. We also conduct an exercise in which the levels of government consumption, government bonds, and net imports, instead of their levels relative to output, are fixed at those under the benchmark tax rate. This case uses the same parameter values as those for the benchmark cases.

Figure 6 shows the consumption Gini and the standard deviation of log of consumption. Since the levels of the measures substantially differ between the case of less persistent risk and others, we show them in two separate panels. For all the cases with divisible labor, increasing transfers financed through consumption taxes substantially decreases consumption inequality and uncertainty. In contrast, when labor is indivisible, expanding consumption-tax financed transfers does not reduce the measures as much as when labor is divisible. When idiosyncratic productivity is less persistent (I-IP), consumption inequality and uncertainty actually rise as the consumption tax rate exceeds around 20%. It is true that in the case with large labor hours (I-H), the reduction in consumption inequality and uncertainty is larger than that in the baseline case. For example, the consumption Gini decreases by 0.006 as the tax rate rises from 5% to 35%. Nonetheless, the reduction in the measure is much smaller than that seen in the case of divisible labor.

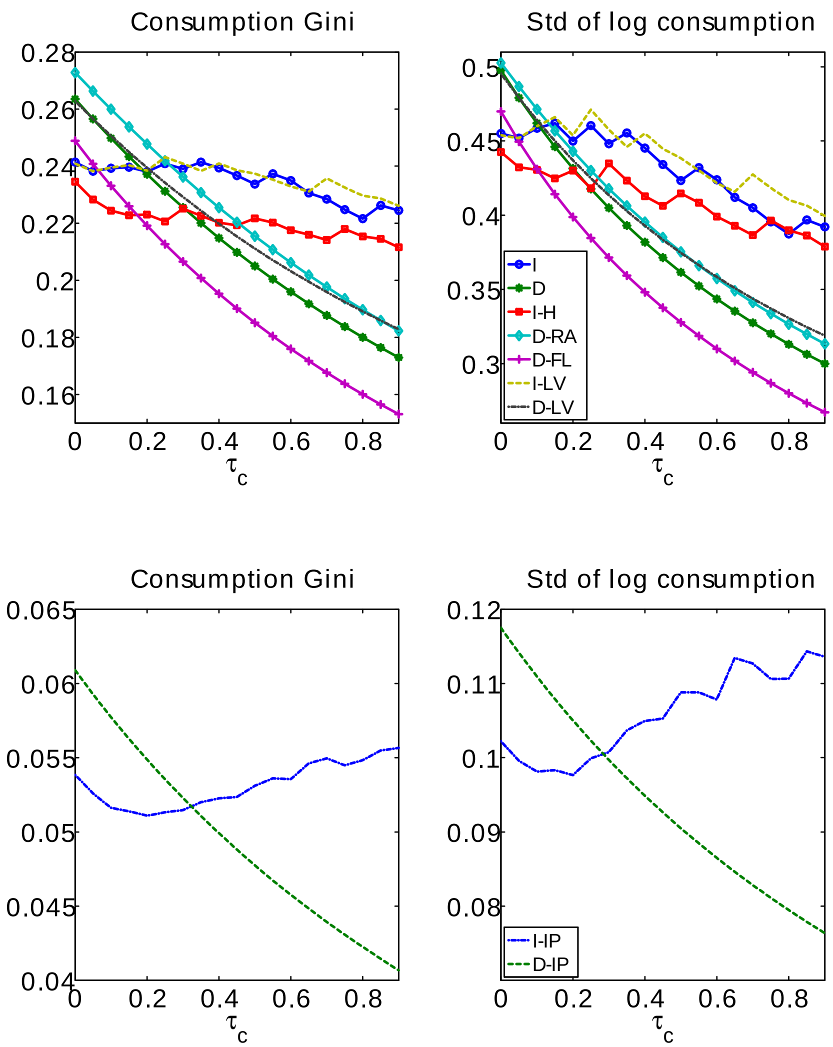

Figure 6: Consumption inequality and uncertainty. I: baseline indivisible labor. D: baseline divisible labor. I-H: I with large hours. D-RA: D with high risk aversion. D-FL: D with low labor supply elasticity. I(D)-IP: I (D) with less persistent productivity. I (D)-LV: I (D) with fixed levels of government consumption, bonds, and net imports.

Figure 7 displays the capital accumulation and wealth inequality. As for divisible labor, in all the cases considered, raising the consumption tax rate and increasing transfers reduce the capital-output ratio and hence raises the interest rate. The wealth Gini rises monotonically. In contrast, when labor is indivisible, the capital-output ratio rises and the interest rate falls with the consumption tax rate, at least initially. The wealth Gini initially falls before rising.

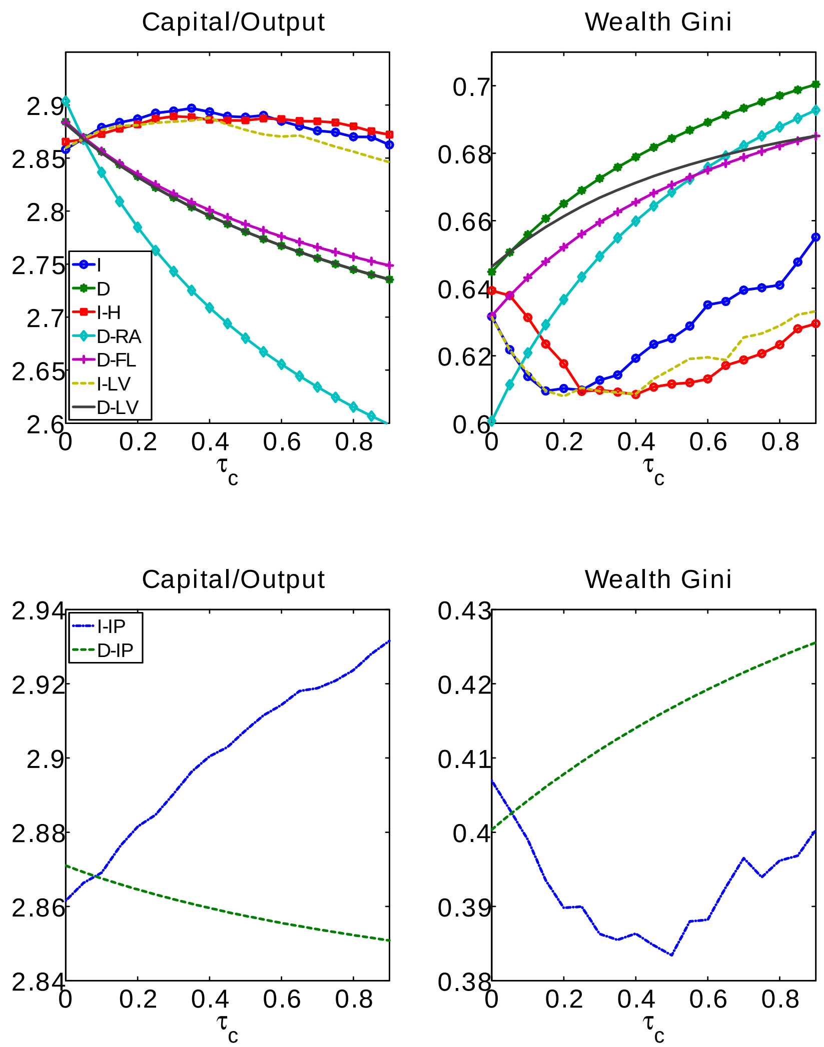

Figure 7: Capital-output ratio and wealth inequality. I: baseline indivisible labor. D: baseline divisible labor. I-H: I with large hours. D-RA: D with high risk aversion. D-FL: D with low labor supply elasticity. I(D)-IP: I (D) with less persistent productivity. I (D)-LV: I (D) with fixed levels of government consumption, bonds, and net imports.

Table 3 lists the optimal consumption tax rate and the welfare gains of raising the tax rate from 5% to 35%.[15] The result shows the robustness of the findings in the previous section. In particular, the sum of inequality and uncertainty gains is positive under divisible labor and negative under indivisible labor. The results in the table are also reasonable. For

[15]The optimal consumption tax rate τ_c is expressed as %. The welfare gains $(\omega_U, \omega_{lev}, \omega_{ine} + \omega_{unc})$ are those of raising the tax rate from 5% to 35% and they are expressed as a percent of the consumption at a 5% tax rate. Recall that the sum of the inequality and uncertainty gains is independent of how we determine the certainty-equivalent leisure.

	Optimal τ_c	ω_U	ω_{lev}	$\omega_{ine} + \omega_{unc}$
D	10	–1.0	–4.3	3.4
D-LV	0	–3.7	–6.5	3.0
D-RA	20	0.7	–6.1	7.2
D-FL	20	–0.2	–3.1	3.4
D-IP	0	–4.5	–5.1	0.7
I	10	–1.2	3.2	–4.2
I-LV	5	–3.7	0.0	–3.6
I-H	15	–0.2	4.2	–4.0
I-IP	0	–4.8	–1.6	–3.3

Table 3: Welfare implication.

example, the level cost is higher when the amount of government consumption rather than its level relative to output is fixed (D-LV versus D and I-LV versus I). The reason is that when the amount is fixed, the level relative to output increases because of lower output, leading to lower consumption and leisure. Hence, the level cost is higher and the optimal consumption tax rate is lower than the benchmark cases.

To summarize, the findings in the present section show the robustness of the results seen in the previous section. Hence, the divisibility of labor is an important factor in determining the impact of consumption-tax financed transfers on consumption inequality and uncertainty as well as on aggregate saving and the wealth distribution.

6 Conclusion

Does increasing public transfers always reduce consumption inequality and uncertainty when households face labor earnings uncertainty? We have studied the question in a heterogeneous-agent model with uninsured wage risk and endogenous labor supply, focusing on transfers financed through consumption taxes. We have calibrated the model to the U.S. economy. We have found that the impact of consumption-tax financed transfers crucially depends on the divisibility of labor. When labor is divisible, raising the consumption tax rate and increasing transfers substantially reduce consumption inequality and uncertainty. In contrast, when labor is indivisible, the effect is much smaller. For example, raising the consumption tax rate from 5% to 35% hardly reduces the measures of consumption inequality and uncertainty in the case of indivisible labor. Indeed, such a policy change worsens inequality and uncertainty in terms of welfare.

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7 Appendix: Labor and Capital Income Taxes

We change the labor and capital income tax rates from 0% to 90% by 5% each independently and adjust transfers accordingly. Unlike consumption taxes, there is a peak in the Laffer curves for labor and capital income taxes and the amounts of transfers decrease as the tax rates exceed a certain level (Figure 8).

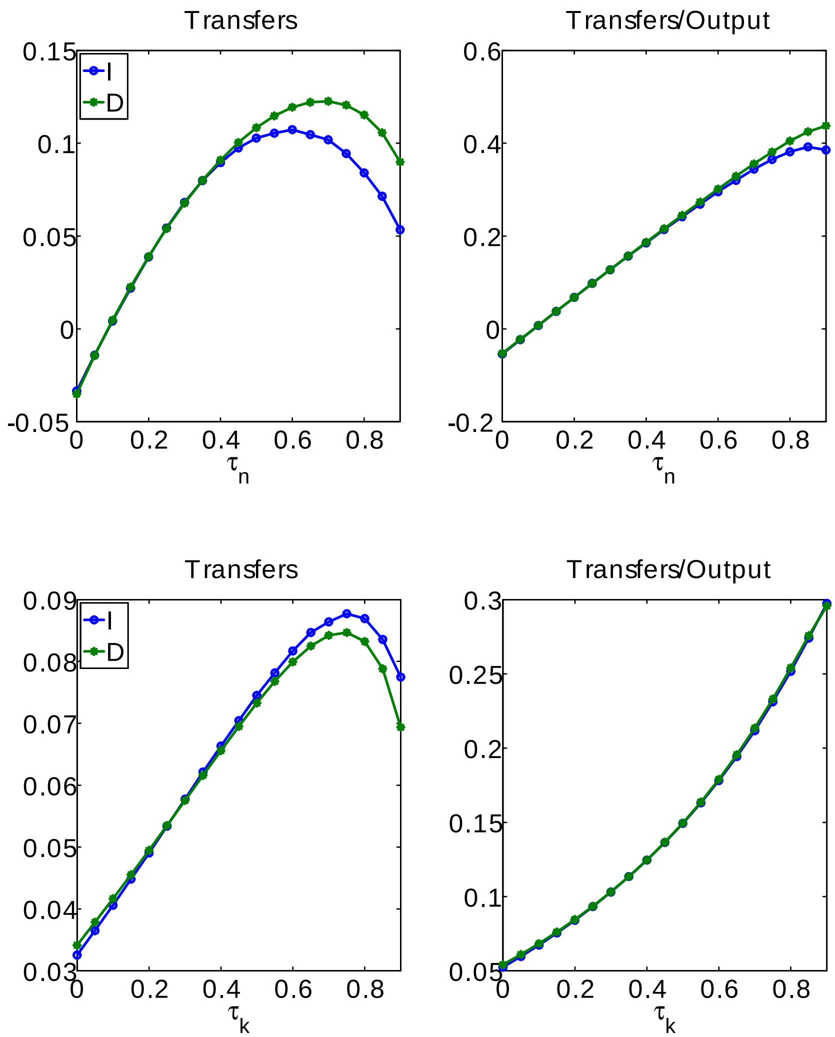

Figure 8: Transfers (Labor and capital income taxes). Upper panel: labor income taxes. Lower panel: capital income taxes. I: baseline indivisible labor. D: baseline divisible labor.

As for labor income taxes, consumption inequality decreases with the tax rate under both divisible and indivisible labor, as shown in Figure 9. The result is in line with the findings of previous works, such as Flodén (2001) (divisible labor) and Alonso-Ortiz and Rogerson (2010) (indivisible labor). In the case of labor income taxes, a hike in the tax rate substantially decreases the after-tax wage rate. Hence, even in the case of indivisible labor, the dispersion of total income significantly decreases with labor income taxes and transfers, leading to large reduction in consumption inequality.

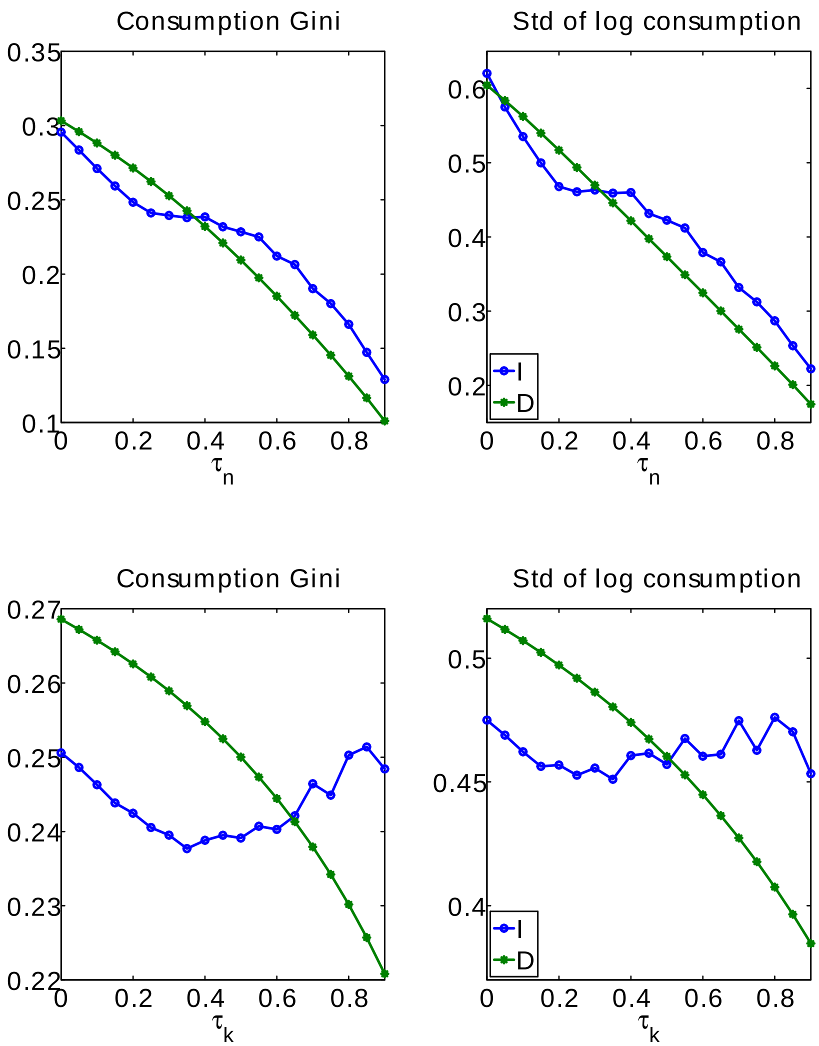

Figure 9: Consumption inequality and uncertainty (Labor and capital income taxes). Upper panel: labor income taxes. Lower panel: capital income taxes. I: baseline indivisible labor. D: baseline divisible labor.

However, the changes in consumption inequality are different under divisible and indivisible labor. Specifically, as the tax rate rises from 0%, the reduction in consumption inequality is initially larger under indivisible than divisible labor. Since the level of transfers is extremely low, low-wealth, low-productivity households stay employed and work for the same hours as before under indivisible labor, which increases their total income. In contrast, those households somewhat reduce labor hours under divisible labor. As the tax rate reaches around 25%, however, further increases in the tax rate and transfers reduce the employment of low-wealth, low-productivity households under indivisible labor and with a relatively low level of transfers, the number of low-income households increases. Hence, the reduction in consumption inequality slows. As the tax rate exceeds around 40%, larger transfers increase the total income of low-wealth and nonemployment households, reducing consumption inequality.

The welfare analysis shown in Figure 10 confirms these results.[16] Under divisible labor, increasing labor income taxes and transfers always increases the sum of the inequality and uncertainty gains. Under indivisible labor, such a policy improves inequality and uncertainty in terms of welfare only when the tax rate is low or high. For example, expanding transfers worsens inequality and uncertainty when the tax rate is raised from the current rate of 28%. The optimal tax rate is 30% for both divisible and indivisible labor.

As for capital income taxes, consumption inequality decreases with the tax rate and transfers under divisible labor, but not monotonically under indivisible labor. Under divisible labor, all households reduce labor hours with larger transfers, and hence the dispersion of total income decreases, reducing consumption inequality. Even under indivisible labor, increasing the tax rate from 0% initially decreases consumption inequality. As in the case of labor income taxes, since the level of transfers is still low, low-wealth, low-productivity households remain employed, increasing their total income and reducing consumption inequality. As the tax rate exceeds around 20%, however, those households switch to nonemployment, while wealth-rich, high-productivity households keep working for the same hours as before

[16] We only present the result when we set $h = hopt$ in determining the certainty-equivalent leisure. As shown earlier, only the decomposition between the inequality and uncertainty gains changes when setting $h = H$.

under a higher wage. Notice that the increase in transfers is relatively small compared to the case of consumption taxes. Hence, consumption inequality increases under indivisible labor.

The welfare analysis suggests the same results. When labor is divisible, raising the capital income tax rate and increasing transfers improve inequality and uncertainty in terms of welfare. Under indivisible labor, such a policy improves inequality and uncertainty only when the tax rate is low. The optimal tax rate is 5% under divisible labor, while it is 20% under indivisible labor.

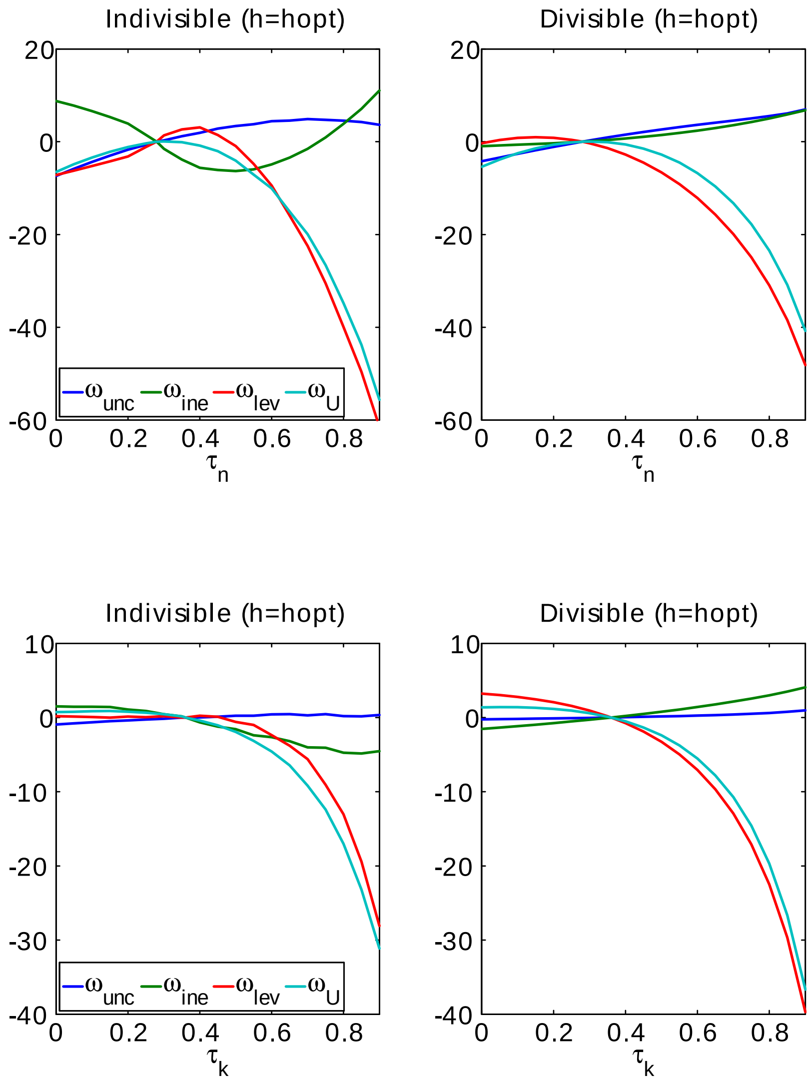

Figure 10: Welfare (Labor and capital income taxes). The horizontal axis shows the welfare gain, which is expressed as a percent of consumption at the initial tax rate ($\tau_n = 0.28$ and $\tau_k = 0.36$). We determine the certainty-equivalent leisure by setting hours to the current level ($h = hopt$).

8 Appendix: Consumption Taxes in Japan

In this appendix, we analyze how increasing consumption-tax financed transfers affects consumption inequality and uncertainty in Japan. The exercise serves as an additional robustness check for the results shown in the main text. Further, it is of interest because a hike in the consumption tax rate is planned in Japan.[17]

8.1 Parameter Values

Parameter values are mostly taken from Nutahara (2015), who determines those values based on previous works, such as Hayashi and Prescott (2002), Sugo and Ueda (2008), and Gunji and Miyazaki (2011). The capital depreciation rate δ is 0.06 and the capital share θ is 0.37. The share of government consumption in GDP γ is 0.154. The capital tax rate τ_k is 52%, while the labor tax rate τ_k is 29%. The net import m is –1.6% of GDP. The government debt b is 111% of GDP. Two departures from Nutahara (2015) are the consumption tax rate and the growth rate of real GDP. We set their values to the average values between 1995 and 2013: The consumption tax rate τ_c is 5%, while the growth rate g is 1.0% per year.[18]

As for the utility function, we set $\bar{h} = 0.333$ for the economy with indivisible labor, as in the U.S. case. We then choose the disutility parameter ψ so that total labor hours H is 0.212, as in Nutahara (2015). For the economy with divisible labor, we set $\mu = 1.0$ in order to keep the comparability with the indivisible labor economy. We set $\varphi = 1.0$ and adjust κ so that the total labor hours is $H = 0.212$.

The stochastic process for idiosyncratic productivity is typically estimated using panel data of individual wages. Since such panel data is limited in Japan, we use the same values for the U.S. economy: $\rho = 0.94$ and $\sigma = 0.205$.[19] The AR(1) process is approximated with a Markov chain as explained in Section 3.

[17] The consumption tax rate rose from 5% to 8% in April 2014 and it will rise to 10% in April 2017.

[18] In contrast, Nutahara (2015) sets the growth rate to 2.1%, which is the average growth between 1980 and 2009, and he sets the consumption tax rate to 10%, which is the rate after March 2017.

[19] The calibrated model generates the wealth Gini similar to that in Japan. See Lise, Sudo, Suzuki, Yamada, and Yamada (2014).

	divisible D	indivisible I
β	0.9829	0.9802
g	0.01	
μ	1.0	NA
φ	1.0	
κ	6.75	NA
H	0.212	
$\bar{h}$	NA	0.333
ψ	NA	7.31
γ	0.154	
δ	0.06	
θ	0.37	
τ_c	0.05	
τ_n	0.29	
τ_k	0.52	
m	–0.016	
ρ	0.94	
σ_ε	0.205	
μ_ε	–0.01162	
b	1.11	

Table 4: Parameter values (Japan).

Lastly, we choose the discount factor β so that the after-tax rate of return on savings r is 2.06% at the benchmark parameter values, following Nutahara (2015). The result is $\beta = 0.9829$ for the economy with divisible labor and $\beta = 0.9802$ for the economy with indivisible labor. All the parameter values are listed in Table 4.

8.2 Results

We conduct the same exercise in Section 4. Figures 11–13 show the results.[20] As in the U.S. case, increasing consumption-tax financed transfers is effective in reducing consumption inequality and uncertainty under divisible labor, but it is not very much under indivisible labor. One difference from the U.S. results is that in Japan, the sum of the inequality and uncertainty gains becomes positive even under indivisible labor when the consumption tax

[20] The optimal consumption tax rate is 5% under divisible labor, while it is 10% under indivisible labor.

rate is raised above 45%. Nonetheless, as in the U.S. case, those gains are substantially smaller than those achieved under divisible labor.

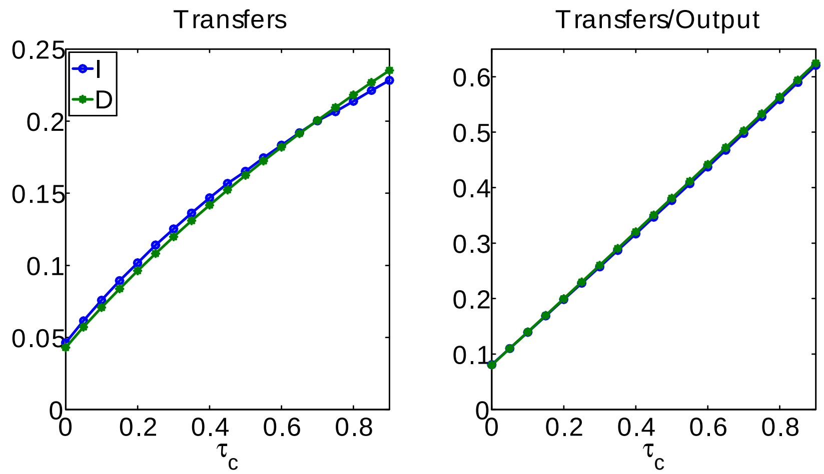

Figure 11: Transfers (Japan). I: baseline indivisible labor. D: baseline divisible labor.

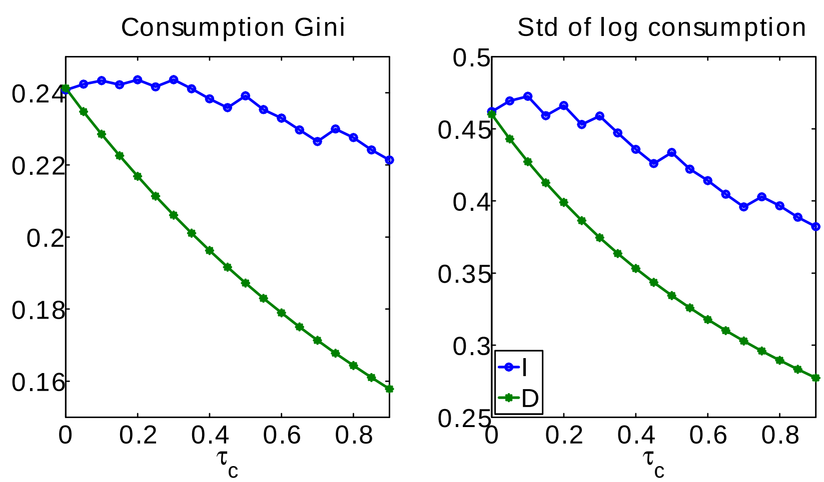

Figure 12: Consumption inequality and uncertainty (Japan). I: baseline indivisible labor. D: baseline divisible labor.

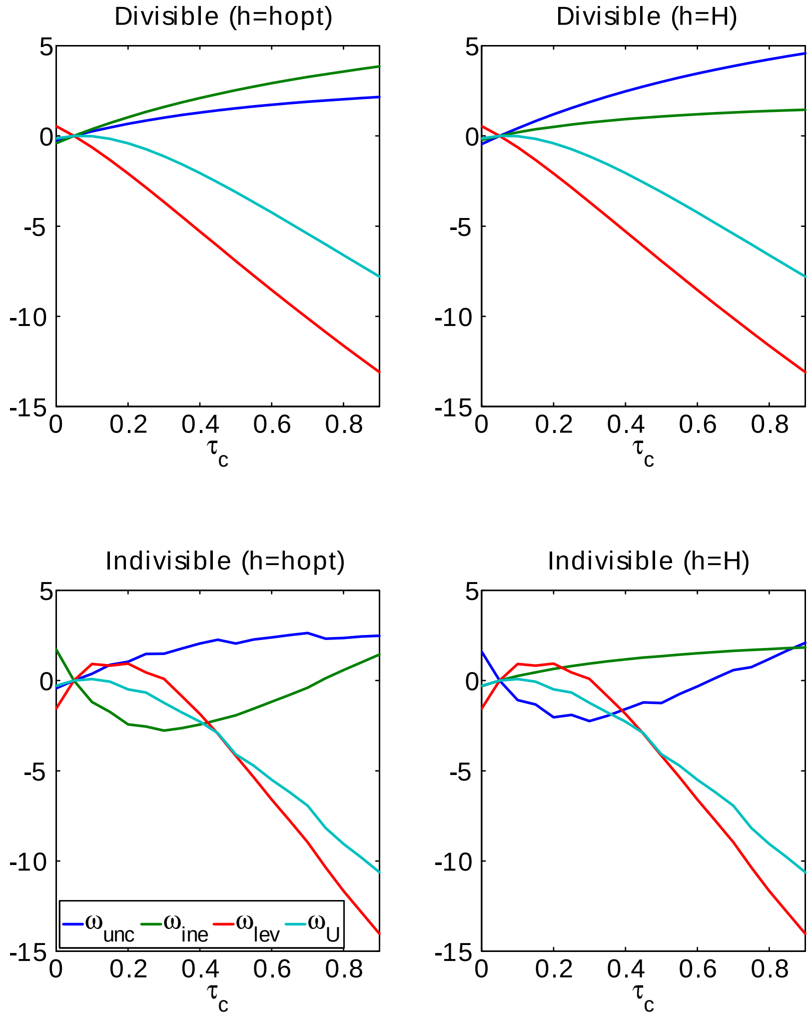

Figure 13: Welfare (Japan). The horizontal axis shows the welfare gain, which is expressed as a percent of consumption at the initial tax rate ($\tau_c = 0.05$). We determine the certainty-equivalent leisure in two ways: setting hours to the current level ($h = hopt$) and to the economy average ($h = H$).