JSPS Grants-in-Aid for Scientific Research (S)

Understanding Persistent Deflation in Japan

Working Paper Series

No. 084

May 2016

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A Double Counting Problem in the Theory of Rational Bubbles

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May 2, 2016

Abstract

In a standard overlapping generations model, the unique equilibrium price of a Lucas' tree can be decomposed into the present discounted value of dividends and the stationary monetary equilibrium price of fiat money, the latter of which is a rational bubble. Thus, the standard interpretation of a rational bubble as the speculative component in an asset price double-counts the value of pure liquidity that is already part of the fundamental price of an interest-bearing asset.

Keywords: rational bubbles; liquidity.
JEL classification: E3.

*I thank Akihisa Shibata for helpful discussion of the paper. I am also grateful for financial support from JSPS Grant-in-Aid for Scientific Research (24223003).

1 Introduction

A rational bubble is usually modeled as an intrinsically useless asset. As shown by Tirole (1985), it attains a positive market value if it is expected to be exchangeable for goods in the future. It becomes worthless if it is expected to be worthless in the future, given that it has no intrinsic use. This property of self-fulfilling multiple equilibria has been used to explain a large boom-bust cycle in an asset price, as a stochastic transition between the two equilibria can generate a boom-bust cycle without any associated change in asset fundamentals.

In this result, a rational bubble is interpreted as the speculative component in an asset price, as typical assets that show a boom-bust cycle, such as land and stock, have some intrinsic use, or yield returns. Thus, it is implicitly assumed that an asset price is the sum of a fundamental component and a rational bubble. This paper shows that this interpretation of a rational bubble is invalid, as it double-counts the value of pure liquidity that is already part of the fundamental price of an interest-bearing asset. This result is due to a basic property of a competitive market such that an asset always attains a positive price, and hence liquidity, as long as it yields a positive dividend. Thus, the value of pure liquidity, if any, is always part of the price of an interest-bearing asset. This paper demonstrates this result by decomposing the price of a Lucas' tree in a standard overlapping generations model.

A recent strand of literature, such as Kocherlakota (2008) and Miao and Wang (2014), introduces an independent autoregressive component into the equilibrium price of an interest-bearing asset by considering borrowing constraints loosened by the component. This paper's result is complementary to their works, as the result implies that it is necessary to incorporate an interest-bearing asset explicitly into a model to identify an environment in which speculation can cause a boom-bust cycle in an asset price.

2 The model

Time is discrete and indexed by $t = 0, 1, 2, ...$ A unit continuum of agents are born in each period and live for two periods. Call agents in their first period "young", and those in their second period "old".

Each agent maximizes the following utility function:

$$U = \ln c_{1,t} + \beta \ln c_{2,t+1}, \tag{1}$$

where $\beta \in (0, 1)$, and $c_{1,t}$ and $c_{2,t}$ denote the consumption of goods by a young and an old agent, respectively, in period t. Each agent is endowed with amounts e_Y and e_O of perishable goods when young and old, respectively. Assume that

$$\beta e_Y > e_O > 0. \tag{2}$$

There also exist a unit continuum of the initial old in period 0 who maximize their consumption in the period and exit from the economy after that. The initial old do not have any good, but are endowed with a share of a Lucas' tree for each. A Lucas' tree is divisible, never depreciates, and yields an amount d (≥ 0) of goods as a dividend at the beginning of each period. There exists a competitive market for the shares of a Lucas' tree in each period.

Under these assumptions, the maximization problem for a young agent born in period t is specified as:

$$\max_{c_{1,t}, c_{2,t+1}, a_t} \ \ln c_{1,t} + \beta \ln c_{2,t+1}, \tag{3}$$

$$\text{s.t.} \ c_{1,t} = e_Y - q_t a_t, \tag{4}$$

$$c_{2,t+1} = e_O + (q_{t+1} + d) a_t, \tag{5}$$

$$c_{1,t}, c_{2,t+1}, a_t \geq 0, \tag{6}$$

where q_t denotes the price of a Lucas' tree in terms of goods in period t, and a_t denotes the share of a Lucas' tree held by a young agent at the end of period t. The initial old's consumption of goods in period 0 is $q_0 + d$. Given the values of parameters, $(\beta,\ e_Y,\ e_O,\ d)$, an equilibrium is characterized by the solution for the utility maximization problem for each agent and the value of q_t satisfying the market clearing condition,

$$a_t = 1 \tag{7}$$

for $t = 0, 1, 2...$, where the right-hand side is the fixed-supply of a Lucas' tree.

2.1 Equilibrium value of fiat money

A Lucas' tree becomes fiat money if the dividend, d, is set to zero. As well known in the literature, there exist two stationary equilibria under the assumption of dynamic inefficiency, (2), in this case. One is a non-monetary equilibrium in which $q_t = 0$ for all t. The other is a monetary equilibrium in which $q_t > 0$ for all t. In the latter equilibrium, the first-order condition for a_t implies

$$q_t = \frac{\beta c_{1,t} q_{t+1}}{c_{2,t+1}}. \tag{8}$$

Given $d = 0$, substituting (4), (5), and (7) into (8) yields

$$q_t = p^* \equiv \frac{\beta e_Y - e_O}{1 + \beta} \tag{9}$$

for all t in the stationary monetary equilibrium, where p^* denotes the stationary monetary equilibrium price of fiat money. The assumption (2) ensures that the right-hand side of (9) is positive.

The positive value of fiat money in a monetary equilibrium is a rational bubble, because it stems from self-fulfilling expectations of the resellability of fiat money for goods in the future. Thus, a rational bubble represents the value of pure liquidity.

2.2 Equilibrium price of a Lucas' tree with a positive dividend

Next, suppose that $d > 0$. In this case,

$$q_t > 0 \tag{10}$$

for all t in any equilibrium, as otherwise agents would demand an infinite share of a Lucas' tree in the competitive market. The first-order condition for a_t is

$$q_t = \frac{\beta c_{1,t}(q_{t+1} + d)}{c_{2,t+1}}. \tag{11}$$

Then, substituting (4), (5), and (7) into (11) yields

$$q_{t+1} = \frac{e_O q_t}{\beta e_Y - (1+\beta) q_t} - d \tag{12}$$

for $t = 0, 1, 2...$

Figure 1 draws the phase diagram implied by (12). The point E is the unique equilibrium, because q_t becomes negative in some period t to satisfy (12) unless q_0 takes the stationary equilibrium value at the point E. Even though the figure is a numerical example, this result of unique equilibrium holds for any parameter values.[1]

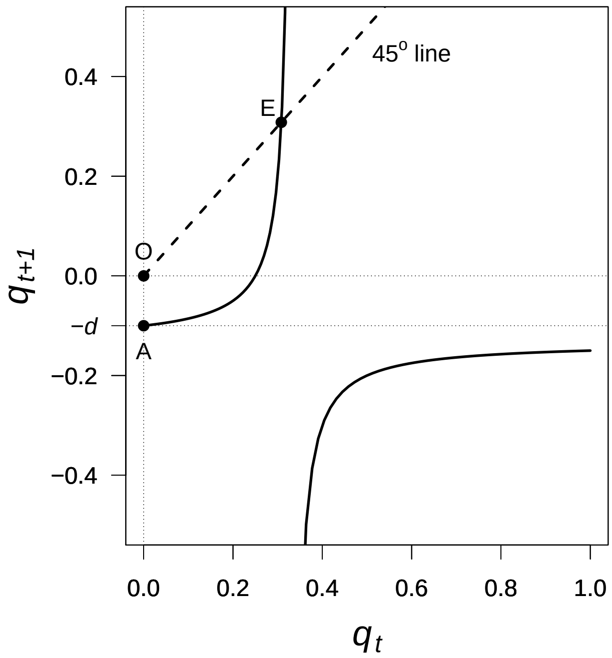
This result is essentially the same as the results shown by Homburg (1991) and Rhee (1991) on the dynamic efficiency of an economy with land. The novel contribution of this paper is in the next part, which shows that the value of pure liquidity represented by a rational bubble is in fact part of the unique equilibrium price of a Lucas' tree.

2.3 Decomposition of the equilibrium price of a Lucas' tree

The uniqueness of equilibrium with a Lucas' tree contrasts with the existence of multiple equilibria with fiat money. This result is due to a bifurcation. The solid curve in Figure 1 moves upward as the dividend on a Lucas' tree, d, declines towards zero. The limit case is the economy with fiat money. In the limit, the point A converges to the origin at the point O, which corresponds to the non-monetary equilibrium with fiat money. Also, the point E converges to some point on the 45-degree line at which q_t and q_{t+1} are positive. This point corresponds to the stationary monetary equilibrium with fiat money. Thus, for $q_t \geq 0$, the

[1] Equation (12) is a hyperbolic function of q_t. For $q_t \in [0, \beta e_Y/(1+\beta))$, (12) is monotonically increasing and strictly convex and has a range over $[-d, \infty)$. For $q_t \in \mathbf{R} \backslash [0, \beta e_Y/(1+\beta))$, q_{t+1} takes a negative value below $-d$.

Figure 1: Phase diagram for the model with a Lucas' tree



Note: $(\beta,\ e_Y,\ e_O,\ d) = (0.5,\ 1,\ 0.05,\ 0.1)$. The solid curve and the dotted line are (12) and a 45-degree line, respectively.

solid curve and the 45-degree line in Figure 1 intersect only once if $d > 0$, but twice if $d = 0$. This bifurcation property of the economy holds because an asset always attains a positive equilibrium price in a competitive market as long as it yields a positive dividend. Hence, unlike fiat money, there is no equilibrium in which a Lucas' tree with a positive dividend loses liquidity.

The convergence of the point E to the stationary monetary equilibrium with fiat money implies

$$\lim_{d \searrow 0} q = p^*, \tag{13}$$

where q denotes the value of q_t and q_{t+1} at the point E. This result indicates that the unique equilibrium price of a Lucas' tree, q, consists of a contribution from positive future dividends, d, and the value of pure liquidity represented by a rational bubble, p^*.

To confirm this result formally, decompose q using (8) and (11):

$$\begin{aligned} q - p^* &= \Lambda(q + d) - \Lambda^* p^*, \\ &= \Lambda(q + d - p^*) - (\Lambda^* - \Lambda)p^*, \\ &= \frac{\Lambda d - (\Lambda^* - \Lambda)p^*}{1 - \Lambda}, \end{aligned} \tag{14}$$

where Λ and Λ^* denote the discount factors, $\beta c_{1,t}/c_{2,t+1}$, in the equilibrium with a Lucas' tree and the stationary monetary equilibrium with fiat money, respectively:

$$\Lambda \equiv \frac{\beta(e_Y - q)}{e_O + q + d}, \tag{15}$$

$$\Lambda^* \equiv \frac{\beta(e_Y - p^*)}{e_O + p^*}. \tag{16}$$

Rearranging both sides of (14) yields

$$\rho q = d + (1 + \rho)\frac{\rho^* p^*}{1 + \rho^*}, \tag{17}$$

where ρ and ρ^* denote the time preference rates implied by Λ and Λ^*, respectively:

$$\rho \equiv \frac{1-\Lambda}{\Lambda}, \tag{18}$$

$$\rho^* \equiv \frac{1-\Lambda^*}{\Lambda^*}. \tag{19}$$

The left-hand side of (17), ρq, is the yield on a Lucas' tree with the time preference rate, ρ, as the discount factor. Thus, (17) is a standard asset-pricing formula. On the right-hand side of (17), the first term, d, is the physical dividend on a Lucas' tree. In the second term, $\rho^* p^*/(1+\rho^*)$ is the present discounted value of the internal yield on fiat money, $\rho^* p^*$, in the stationary monetary equilibrium. Hence, the yield on a Lucas' tree consists of the physical dividend and the value of pure liquidity represented by a rational bubble. The second term on the right-hand side of (17) is multiplied by the gross time preference rate in the economy with a Lucas tree, $1+\rho$, to convert it into the current value.[2]

The decomposition formula, (17), is valid even for a dynamically efficient economy, as p^* is simply zero in that case. Thus, the proper definition of the fundamental price of an interest-bearing asset must include the value of pure liquidity given the underlying environment in the economy. This result is a reflection of an earlier result that an asset with a positive dividend always attains a positive price, and hence liquidity, in a competitive market. It in turn implies that it is a double-counting of the value of pure liquidity if an asset price is assumed to be the sum of the fundamental price and a rational bubble on an intrinsically useless asset, so that the latter component can be switched on and off exogenously.

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