

# Product Cycle and Prices: a Search Foundation

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# Product Cycle and Prices: a Search Foundation\*

Mei Dong<sup>†</sup>      Toshiaki Shoji<sup>‡</sup>      Yuki Teranishi<sup>§</sup>

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## Abstract

This paper develops a price model with a product cycle. Through a frictional product market with search and matching frictions, an endogenous product cycle is accompanied with a price cycle where a price for a new good and a price for an existing good are set in a different manner. This model nests a New Keynesian Phillips curve with the Calvo's price adjustment as a special case and generates several new phenomena. Our simple model captures observed facts in Japanese product level data such as the pro-cyclicality among product entry, demand, and price. In a general equilibrium model, an endogenous product entry increases variation of the inflation rate by 20 percent in Japan. This number increases to 72 percent with a price discounting after a first price.

*Keywords:*    Phillips curve; product and price cycles; search and matching

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# 1 Introduction

“We have all visited several stores to check prices and/or to find the right item or the right size. Similarly, it can take time and effort for a worker to find a suitable job with suitable pay and for employers to receive and evaluate applications for job openings. Search theory explores the workings of markets once facts such as these are incorporated into the analysis. Adequate analysis of market frictions needs to consider how reactions to frictions change the overall economic environment: not only do frictions change incentives for buyers and sellers, but the responses to the changed incentives also alter the economic environment for all the participants in the market. Because of these feedback effects, seemingly small frictions can have large effects on outcomes.”

Peter Diamond

“Price dynamics in imperfectly competitive markets result from the interplay of sellers’ and buyers’ strategies. Understanding the microeconomic determinants of price setting and their welfare or macroeconomic implication - such as the role of friction in monopolistic competition or the effects of inflation - therefore requires an analysis which incorporates the decision problems of both types agents.”

Roland Benabou

Recent observations from micro data reveal facts for product cycles. These data also show that a product cycle is accompanied with a price cycle.

Broda and Weinstein (2010) find that using the universe of products data, the product turnover rate in the United States is about 25 percent annually. They uncover that these product cycles have a significant effect on the aggregate price index. Nakamura and Steinsson (2012) shed light on product turnover being a key mechanism for price change using micro data for trade price indexes, a discussion of so called a product replacement bias. A first price holds a non-trivial effect on price dynamics. They also show that 40 percent of products are replaced without any price change after the introduction of goods into markets. Thus, a first price and subsequent prices for a product behave differently, which forms a price cycle. In this simple case, a price cycle implies that a first price is flexibly set to optimal level and subsequent prices do not change under a product cycle. Ueda, Watanabe, and Watanabe (2016) confirm the same facts for Japan using the point

of sale scanner data of retail goods. They reveal that the product turnover rate is 30 percent annually. Price adjustment occurs in time of product turnover and more than half of products do not experience price changes until their exits from the market.<sup>1</sup> On average, they show that a product price declines after a first price and a price increases in a time of introducing a new product into a market. This is another case of a price cycle under a product cycle.<sup>2</sup>

To explain price dynamics, we have a lot of former studies with different concepts and specifications. A large number of papers for the New Keynesian Phillips curve assumes the Calvo (1983) - Yun (1996) price adjustment in which firms optimally change prices with a certain probability. Their price adjustment mechanism provides a useful proxy for a price stickiness.<sup>3</sup> Golosov and Lucas (2007) set up a menu cost model in which a price is changed when firm can pay a real menu cost under idiosyncratic productivity shocks and general inflation. Their model explains a mechanism behind the New Keynesian Phillips curve based on exogenous probability of a price change.<sup>4</sup> Mankiw and Reis (2002) build up a sticky-information model. They assume that information diffusion is slow and information updating makes it costly to reset a goods price.

In sharp contrast to these studies, we propose a new model to explain price dynamics observed in micro data. We explicitly model entry and exit in a frictional goods market

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<sup>1</sup>For Japanese data, Abe et al. (2017) show that first prices hold a significant effect on a price index for daily necessities and foods in Japan.

<sup>2</sup>Abe et al. (2016) show a price decline after a first price using data of the most popular price comparison website in Japan. Their data includes home electrical appliances and digital consumer electronics.

<sup>3</sup>Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) show that this New Keynesian Phillips curve based on Calvo (1983) - Yun (1996) price adjustment fits macro data well.

<sup>4</sup>Gertler and Leahy (2008) develop a tractable state dependent Phillips curve in contrast to a time dependent Phillips curve based on the Calvo mechanism. They assume that firms being in a state to get benefit over cost optimally reset a new price. This Phillips curve with an Ss foundation has the same form as the New Keynesian Phillips curve. Only a difference between two Phillips curves is a larger response to demand reflecting greater flexibility of price adjustment in the Phillips curve with an Ss foundation. Also, Woodford (2009) shows a similarity and difference between a state dependent pricing model and a time dependent pricing model under limited information.

where two groups of firms, e.x., intermediate goods producers and final goods producers, search for each other. Each new match is considered as a new product. We endogenize entry decisions and leave exit as exogenous. Final goods producers decide to enter into the market when the benefit from selling the new good can cover the cost of entering the market. If a final good producer successfully finds an intermediate good producer, a new good can be produced. Thus, the number of goods in the market varies according to business cycles.

In addition, we assume that firms set a new price upon matching and the subsequent prices in the match follow an exogenous path given the first price. For example, we consider two cases in the simple model: a fixed price after setting the first price and a discounting price at a constant discount rate. In a general equilibrium model, we introduce endogenous price discounting by reflecting depreciation of quality/preference. As we define an aggregate price index which includes both new prices and existing prices, the aggregate price responds to business cycles through an extensive margin effect and an intensive margin effect. For example, when the economy is hit by a positive demand shock, more final goods producers will enter the market which leads to more matches in the goods market. This makes the aggregate price more flexible thanks to more new prices. Moreover, in each new match, the positive demand shock raises total trading surplus which leads to a higher new price. Overall, both entry and prices can be positively correlated with demand. Generally, our model can generate rich price dynamics.

For quantitatively analysis, we use the Japanese Nikkei data to calibrate our model's key parameters. The simulation results show that our model can replicate the observed facts in the data related to product cycles and prices. Overall, the model with price discounting performs the best because it builds in the price discounting feature that is prevalent in the Nikkei data. We also extend the simple partial equilibrium model to a general equilibrium model where final goods producers sell their products to households in a monopolistic competition market. In the general equilibrium model, prices faced by households correspond closely to prices in our data. We find that an endogenous product entry in a frictional goods market can increase the standard deviation of the inflation rate

by 20 percent. This number increases to 72 percent with a price discounting. This result confirms the importance of endogenizing product cycles and price cycles in understanding price dynamics.

Several studies have considered goods market with search and matching frictions.<sup>5</sup> Michaillat and Saez (2015) assume a search and matching goods market. They show that the productive capacity is idle in the U.S. and such no-full operating rate implies that sellers face search frictions to find buyers. They match the model to data and show that a fixed-price model describes the data better than a flexible price model does. Petrosky-Nadeau and Wasmer (2015) develop a DSGE model with search and matching frictions in credit, labor and goods market. Their main goal is to understand how frictions in these markets affect labor market dynamics in response to productivity shocks. Bai et al. (2017) consider a frictional goods market and argue that demand shocks that induce more search can increase output. Their quantitative results show that demand shocks can explain a large share of business cycle fluctuations. All these paper with frictional goods market, however, do not focus on the role of product cycles on price dynamics.

Our paper is more closely related to Bilbiie, Ghironi, and Melitz (2007), which assume an endogenous producer entry in a goods market and a sticky price mechanism following a price adjustment cost of Rotemberg (1982). They derive a Phillips curve including an adjustment cost parameter and the number of goods in the market. The mechanism for price dynamics is, however, sharply in contrast to the one in our paper. There is no matching friction in their model. Their Phillips curve includes the number of products originally from a price index, i.e., variety effect, and an output aggregation. In our model, goods market friction itself introduces a new mechanism in Phillips curve. Our interests are on the role of product cycles on price dynamics in a frictional goods market.

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<sup>5</sup>In the international trade literature, Drozd and Nosal (2012) introduce search and matching frictions into goods trading between two countries to solve puzzles regarding the correlation between prices of real exports and imports and volatilities of the real exchange rate. Eaton, Jinkins, Tybout, and Xu (2016) assume a search and matching process for international buyer-seller connections to explain various empirical issues. These papers support the validity of embedding search and matching into a goods market.

Empirical studies, such as Barrot and Sauvagnat (2016), show that there exist search and matching frictions in production networks using firm level data. They find that the occurrence of natural disasters on suppliers reduces output to their customers when these suppliers produce specific input goods. This implies that specific input goods are not traded in a centralized market that does not need search frictions. Carvalho, Nirei, Saito, and Tahbaz-Salehi (2016) also show that individual firms can not quickly find suitable alternatives under a decentralized goods market with search friction when firms are faced with a supply-chain disruption by a natural disaster in Japan.

The rest of our paper is organized as follows. Section 2 describes our data and the main empirical observations. Section 3 introduces the simple partial equilibrium model with product cycles. We compare three versions of the simple model by inspecting the corresponding New Keynesian Phillips curve in Section 4. Section 5 provides quantitative analysis using Nikkei data. The general equilibrium version of our model is discussed in Section 6. We consider a robustness check of our model in Section 7. Finally, Section 8 concludes.

## 2 Data and Observations

We use the POS scanner data of Nikkei.<sup>6</sup> Our data includes information for prices and quantities in sales for each product at each retail shop on each day from April 1988 to December 2017.<sup>7</sup> The retail shops in our data set consist of supermarkets in Japan, where typically food products and daily necessities are sold. The number of supermarkets during the sample period is more than 200 and increases to 300 in the end of sample. In the basic analysis, we restrict our sample for only supermarkets that are available for all periods. So we use data from 11 supermarkets to exclude a bias by shop bankruptcy.

A barcode including the Japanese Article Number (JAN) code is printed on each of

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<sup>6</sup>See Appendix A for details of data.

<sup>7</sup>We exclude special sales from data. We also exclude the sample from 1994Q4 to 1995Q3 since price change are extremely large on a quarterly base.

these products and products are distinguished by fairly detailed classifications.<sup>8</sup> In addition, barcodes provide information about the product category (such as butter, yogurt, or shampoo) and the manufacturer of each product. Data includes 860,000 products in total, 100,000 products as an average per year, and 30,000 products per retailer per year. Our scanner data cover 170 of the 588 items in the CPI in Japan.<sup>9</sup>

An advantage of this data is that we can observe product cycles, i.e., entry and exit of an individual product, and their effects on prices at the supermarket level in a long sample period covering several business cycles in Japan. We can also uncover price cycles, i.e., price dynamics between entry and exit. This is a new aspect to think of price determination. A first price shows a different behavior from following prices. This rejects the idea of flexible price setting in which a new price and an average price coincide. Moreover, due to product entry and exit, weights of products in aggregating individual prices to an aggregate price sufficiently changes.

**Observation 1: A product cycle is about 9 quarters. Product entry is more volatile than product exit. Standard deviation of the entry rate is 0.023 and that of the exit rate is 0.012.**

Figure 1 shows the entry rate and the exit rate over the sample period. We observe that the entry rate and the exit rate vary according to business cycles. The entry rate is calculated as the number of newly introduced products in a given quarter divided by the total number of products in that quarter. The exit rate is calculated as the number of exiting products in the previous quarter divided by the total number of products in the previous quarter. These entry and exit rates imply that all products are replaced by about 9 quarters.

Table 1 shows basic statistics of product entry and exit and prices in on a quarterly

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<sup>8</sup>In JAN code, the first seven digits of the JAN code indicate the company code and the last six digits indicate the individual product. When JAN codes are different for the same type of products by the same company, these goods are different in such as package and ingredient.

<sup>9</sup>Our scanner data does not include fresh food, recreational durable goods, such as computers and cell phones, and services, such as housing rent and utilities.



base. The first two rows in the table reveal that product entry rather than product exit plays a more important role for prices in business cycles. The entry rate taken as an average of entry rates of all products is 0.12 and its standard deviation is 0.023. On the other hand, the exit rate taken as the average of exit rates of all products is 0.11 and its standard deviation is 0.012. The variation of the entry rate is much larger than that of the exit rate though the average levels of entry and exit rates are almost the same.

**Observation 2: There is pro-cyclicalities between product cycle and price. Moreover, there is pro-cyclicalities between product cycle and demand.**

Table 2 shows correlations among variables.<sup>10</sup> To show robustness of these statistics, we show correlations not only at a quarterly frequency but also at an annual frequency. Data shows pro-cyclicalities between the entry rate and price. The correlation between the entry rate and price is 0.15 at a quarterly frequency and 0.41 at an annual frequency. The entry rate and demand also show pro-cyclicalities. The correlation between the entry rate and demand is 0.12 at a quarterly frequency and 0.48 at an annual frequency. These results are accompanied with pro-cyclicalities between an price and demand. The correlation between prices and demand is 0.87 at a quarterly frequency and 0.8 at an annual frequency.

At the same time, the total number of goods in the market is also related to prices and demand. Data shows pro-cyclicalities between the number of goods and prices/demand. The correlation between the number of goods and prices is 0.72 at a quarterly frequency and 0.79 at an annual frequency. The correlation between the number of goods and demand is 0.81 at a quarter frequency and 0.85 at an annual frequency.

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<sup>10</sup>Note that we exclude outliers on a quarterly base. More specifically, we exclude data of 1994Q4, 1995Q2, and 1995Q3. This exclusion basically decreases the standard deviation of price.

**Observation 3:** The nature of first price clearly differs from those of the following prices. The average of new prices is 38 percent higher than the average price. First prices are set higher than average prices and prices decline thereafter. The ratio of the standard deviation of a first price to that of an average price is 2.33. First prices are flexibly set and prices after first prices gradually decrease on average.

Regarding a first (entry) price and other prices for one specific good, the nature of first price clearly differs from those of the following prices. The third row of Table 1 shows that the average of first prices is 38 percent higher than the average price. This implies that a first price is set higher than an average price and prices decline thereafter. Figure 2 shows how price changes after entry on average. Prices decline after entry.<sup>11</sup> This is consistent with Ueda, Watanabe, and Watanabe (2018). They match successor products and predecessor products in data calculation though they use the same data as ours. In their calculation, the new price of a successor product is about ten percent higher than that of the predecessor product on average. Figure 3 shows the fraction of products whose prices increase, decrease, or do not change over their life span.<sup>12</sup> On average, 23 percent of goods decreases price and 16 percent of goods increases price after entry. Thus, the larger number of goods makes price lower after entry. In the last ten years, the ratio of goods that decreases price increases to 29 percent on average though the ratio of goods that increases price does not change. Such a distortion in price adjustment makes prices decline after entry on average.

The standard deviation of first prices is much larger than that of the average price. Figure 4 shows year to year changes of an average of first prices and an average price. First prices are more volatile than subsequent prices and first prices tend to decide

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<sup>11</sup>We observe the same phenomena of price decline after entry in Melser and Syed (2014), and Abe et al. (2016). Melser and Syed (2014) focus on a large US scanner data set on supermarket products and they find that prices decline as items age on average. Abe et al. (2016) use online price data and show that new product prices decrease gradually after entry and the speed of price decline varies considerably across products.

<sup>12</sup>We can find a similar figure in Ueda, Watanabe, and Watanabe (2018).

movements of average price. The ratio of the standard deviation of first prices to the standard deviation of an average price is 2.33. Prices after entry do not change as much as entry prices. As shown in Figure 3, 61 percent of goods does not change price after entry on average in Japan.<sup>13</sup> No price change after entry contributes to a difference in the standard deviations between new prices and existing prices. Moreover, Ueda, Watanabe, and Watanabe (2016) emphasize a fashion effect for Japanese data. This effect describes that new goods attract higher demand and so have higher prices. Then, prices start to decline by reduction of demand as time goes by. Here, a declining price after entry may induce producers to set a first price sufficiently high. It also contributes to a higher standard deviation of first prices.

The share of products that experience either no price change or a declining price after entry amounts of 84 percent on average in our data. Therefore, embedding these features would be important to understand price cycles in Japan.

### 3 Model with Product Cycles

#### 3.1 Setting

We begin with a simple partial equilibrium model with search frictions in the goods market. There are two types of firms: firm  $A$  and firm  $B$ . Firm  $A$ s and firm  $B$ s trade good  $A$  in a decentralized market. In particular, firm  $A$ s can produce good  $A$ . Firm  $B$ s have demand for good  $A$ , but cannot produce good  $A$ . Therefore, firm  $A$ s and firm  $B$ s randomly search for each other in the decentralized goods market. We can view firm  $A$  as intermediate goods producers and firm  $B$  as final goods producers, where firm  $B$  needs input from firm  $A$  to produce final goods. Firm  $A$  is of measure 1 and firm  $B$  can choose to enter the market with a cost  $\kappa$ .

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<sup>13</sup>This is consistent with Nakamura and Steinsson (2012) that show that 40 percent of products are replaced without price change after the introductions of goods into markets and Ueda, Watanabe, and Watanabe (2016) that confirm that a half of products do not experience price changes until their exit from a market.

Let the measure of unmatched firm  $A$  be  $u_t$  at time  $t$  and the measure of vacant firm  $B$  be  $v_t$ . The matching function exhibits constant return to scale property and is given by

$$m(u_t, v_t) = \chi u_t^{1-\alpha} v_t^\alpha \text{ where } \alpha \in (0, 1). \quad (1)$$

Define the market tightness as  $\theta_t = v_t/u_t$ . The probability for a vacant firm  $B$  to find an unmatched firm  $A$  is denoted as  $s(\theta_t)$  and the probability for an unmatched firm  $A$  to find a vacant firm  $B$  is denoted as  $q(\theta_t)$ , where

$$s(\theta_t) = \frac{m_t}{v_t} = \chi \theta_t^{\alpha-1}, \quad (2)$$

$$q(\theta_t) = \frac{m_t}{u_t} = \chi \theta_t^\alpha. \quad (3)$$

We assume that  $s(0) = 1$  and  $q(\infty) = 1$ . To simplify notations, we use  $(s_t, q_t)$  directly and omit the argument  $\theta_t$  when there is no confusion. Each match is destroyed with an exogenous probability  $\rho \in (0, 1)$ .

Once a firm  $A$  and a firm  $B$  match, the firm  $A$  produces  $Z^A$  units of good  $A$  for firm  $B$  and a new price of good  $A$  is negotiated by the Nash bargaining solution. In the model, we assume that the negotiated price changes by  $g$  from time  $t$  to time  $t+1$  according to the contract during the duration of the match. There is no renegotiation of the price after the new price is determined. New prices are negotiated only when new matches are formed. This infrequent negotiation for price directly follows the spirit of Shimer (2004) and Hall (2005) in a labor search model. For simplicity, the amount of good  $A$  transferred in each match is exogenously given. Moreover, the cost of producing  $Z^A$  units of good  $A$  is  $X_t$ , where  $X_t$  can include any cost of production even though we do not specify the production function at this stage. Changes in  $X_t$  could be interpreted as potential cost push shocks. The benefit for the firm  $B$  to acquire  $Z^A$  units of good  $A$  is  $Z_t^B$ , where  $Z_t^B$  is a nominal variable and includes a random price. This variable works as a demand shock.

The free entry condition for firm  $B$  is

$$\kappa = \beta s_t \mathbb{E}_t V_{t+1} \left( \tilde{P}_{t+1}^A \right), \quad (4)$$

This free entry condition decides the number of a new goods into a market. Thus, product entry and so price setting are endogenous. Firm decides to introduce a new good into a market when a profit from selling a new good with a new price is larger than a cost of introducing it. Trade will take place in the following period, where  $\tilde{P}_{t+1}^A$  denotes the newly negotiated price of good  $A$  and  $V_{t+1}(\cdot)$  denotes the value function for firm  $B$ . Note that there is one period lag for production after a new match, as in the timeline of Trigari (2009).

The value function for a firm  $B$  with a contract of price  $\tilde{P}_t^A$  is

$$V_t(\tilde{P}_t^A) = Z_t^B - Z^A \tilde{P}_t^A + \beta(1 - \rho) \mathbb{E}_t V_{t+1}(g \tilde{P}_t^A), \quad (5)$$

where  $g$  captures changes in the price  $\tilde{P}_t^A$  set at time  $t$  when at time  $t + 1$ . All matches that survive from time  $t$  to time  $t + 1$  are subject to the same price adjust factor  $g$ . The term  $Z_t^B - Z^A \tilde{P}_t^A$  is the flow benefit of being in a match and  $\beta(1 - \rho) \mathbb{E}_t V_{t+1}(g \tilde{P}_t^A)$  shows the continuation value of the match. The new price  $\tilde{P}_t^A$  for goods  $A$  is set by only newly matched firms. The adjustment of price from time  $t$  to time  $t + 1$  is inherent in the contract.

Now consider the value functions for a firm  $A$ . Let  $J_t^1(\tilde{P}_t^A)$  denote the value function for a newly matched firm  $A$  with a negotiated new price  $\tilde{P}_t^A$  at time  $t$ , where

$$J_t^1(\tilde{P}_t^A) = Z^A \tilde{P}_t^A - X_t + \beta \mathbb{E}_t \left[ (1 - \rho) J_{t+1}^1(g \tilde{P}_t^A) + \rho J_{t+1}^0 \right]. \quad (6)$$

The flow benefit of having the match is given by the term  $Z^A \tilde{P}_t^A - X_t$ . If the match survives at time  $t + 1$ , the continuation value is  $J_{t+1}^1(g \tilde{P}_t^A)$ , where  $g$  again indicates the price adjustment within a match. If the match is destroyed at time  $t + 1$ , the firm  $A$  becomes an unmatched one with the value function  $J_{t+1}^0$ . The value of an unmatched firm  $A$  is

$$J_t^0 = \beta \mathbb{E}_t \left[ q_t J_{t+1}^1(\tilde{P}_{t+1}^A) + (1 - q_t) J_{t+1}^0 \right]. \quad (7)$$

For the unmatched firm  $A$ , it can go back to the product market in the same period and find a match with the probability  $q_t$ . Production will take place in the following period and the value for the match is therefore  $\mathbb{E}_t J_{t+1}^1(\tilde{P}_{t+1}^A)$ . With the complementary

probability  $1 - q_t$ , the unmatched firm  $A$  remains unmatched and has the continuation value  $J_{t+1}^0$ . Here the benefit from having a match is  $J_t^1(\tilde{P}_t^A) - J_t^0$ . We can find the value of a new match for firm  $A$  by taking the difference between  $J_t^1(\tilde{P}_t^A)$  and  $J_t^0$ .

In a match, firm  $A$  and firm  $B$  bargain over the price  $\tilde{P}_t^A$  of good  $A$ , taking into consideration that the price is not renegotiated during the duration of the match and the price can adjust by a factor  $g$  from time  $t$  to time  $t + 1$ . The price  $\tilde{P}_t^A$  solves

$$\max_{\tilde{P}_t^A} \left[ V_t(\tilde{P}_t^A) \right]^{1-b} \left[ J_t^1(\tilde{P}_t^A) - J_t^0 \right]^b, \quad (8)$$

where  $b$  is the bargaining power for firm  $A$ . The solution  $\tilde{P}_t^A$  is determined by

$$bV_t^A(\tilde{P}_t^A) = (1 - b) \left[ J_t^1(\tilde{P}_t^A) - J_t^0 \right], \quad (9)$$

Lastly, we describe the flow conditions and the aggregate price index. Following Trigari (2009), a newly separated firm  $A$  can search again in the same period. The measure of unmatched firm  $A$  is

$$u_t = 1 - (1 - \rho) N_t, \quad (10)$$

where  $N_t$  denotes the measure of matches. The flow condition of  $u_t$  is therefore

$$u_{t+1} - u_t = \rho(1 - u_t) - q_t u_t. \quad (11)$$

It follows that

$$N_t = (1 - \rho) N_{t-1} + q_{t-1} u_{t-1}. \quad (12)$$

Since prices in the new matches are set through Nash bargaining and the old prices in survived matches adjust by the factor  $g$  from time  $t$  to time  $t + 1$ , we use an aggregate price index  $P_t^A$  to denote the aggregate price in the economy at time  $t$ ,

$$N_t P_t^A = (1 - \rho) g N_{t-1} P_{t-1}^A + \chi \theta_{t-1}^\alpha u_{t-1} \tilde{P}_t^A. \quad (13)$$

The aggregate price index completes the description of the model, where (2), (3), (4), (5), (6), (7), (9), (10), (12), and (13) are used to solve the model.<sup>14</sup> The inclusion of the

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<sup>14</sup>See Appendix B for more details.

price adjustment factor within a match is motivated by observations from the Japanese data that prices decline after the first price, as shown in Figure 2. We take this declining pattern of the price cycle as given. There are theoretical models that can rationalize this pricing pattern through the fashion effect or the product quality signalling effect. In this case, firms in a match negotiate the first price taking into account of the passive price discounting after the first price. In the steady state, an aggregate price index  $P^A$  from (13) leads to

$$P^A = \frac{\rho}{1 - (1 - \rho)g} \tilde{P}^A, \quad (14)$$

where  $\tilde{P}^A$  is the steady state price of good  $A$  in a new match. Given that  $g \in (0, 1]$ , we have  $P^A < \tilde{P}^A$ . This can generate a difference between the average new prices and the average price as found in the data. The model allows for product entry and exit. Entry decisions are endogenous and depend on the parameters and shocks. So far, exit is exogenous. This setting in a model consistent with Japanese data that shows a sufficiently larger standard deviation of an entry rate than that of an exit rate. The model can generate an endogenous number of products and can be used to examine how product entry is correlated with price and demand. We call this model as a model with price discounting in particular when  $g < 1$ . In the following, we consider two special cases: one with no price discounting and one with exogenous entry.

### **3.2 A Model with Endogenous Entry with $g = 1$ : No Price Change After a First Price**

One special case of the model is to assume that the price is fixed after setting the new price. That is,  $g = 1$  for all  $t$ . In our data, about more than half of goods do not experience price changes after entry. Compared with the sticky price model by Calvo (1983) - Yun (1996) price adjustment in which firms optimally change prices with a certain probability, a new price is set optimally only when a new match is formed in the goods market in our model. There, however, is still variation of the number of products in the market so that an extensive margin effect exists for a price change.

### 3.3 A Model with Exogenous Entry

Another way is to assume that entry into the product market is exogenous. In this way, there is a constant number of product in the goods market in each period. Both the entry probability and exit probability are exogenous, so in some senses we exclude the role of goods market frictions.<sup>15</sup> In this model, there is no extensive margin effect on a price change since the number of products is constant. We also assume  $g = 1$  for all  $t$ . This model has a similar structure with the traditional New Keynesian model by the Calvo (1983) - Yun (1996) sticky price and has very similar dynamics against demand shock as New Keynesian Phillips curve as discussed below. It can serve as a benchmark to facilitate comparison with other models and help us understand the role of goods market frictions.

## 4 Inspecting Three Models

Before we solve our models quantitatively, we highlight the role of an endogenous number of products in the product market by log-linearizing the system of equations around a constant steady state with zero inflation. Linearized price equations are convenient to reveal the features of price dynamics in particular comparing to the New Keynesian Phillips curve by Calvo (1983) - Yun (1996). To keep a fair comparison with the New Keynesian Phillips curve, we use relative price in the value functions in which an individual price set by firms is divided by the aggregate price.<sup>16</sup> We express the log-deviation of a variable (e.g.,  $P_t$ ) from its efficient steady-state value ( $\bar{P}$  or  $P$ ) by placing a hat ( $\hat{\cdot}$ ) over the lower case symbol ( $\hat{p}_t$ ).

For the model with exogenous entry, we have the following linearized price equation

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + b \frac{\rho [1 - \beta(1 - \rho)]}{1 - \rho} \frac{\bar{Z}^B}{Z^A} \hat{Z}_t^B, \quad (15)$$

where the inflation rate is defined as  $\pi_t \equiv \hat{p}_t^A - \hat{p}_{t-1}^A$ .<sup>17</sup> Price dynamics simply depends

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<sup>15</sup>See Appendix C for details of this model.

<sup>16</sup>See details in Appendix D.

<sup>17</sup>To simplify expressions, we assume that a cost shock  $X_t$  is zero and a steady-state aggregate price



on the demand shock  $\hat{Z}_t^B$ . The effect of goods market frictions appears only through  $\rho$  in the coefficient on demand shock. The exit rate  $\rho$  works as a probability of re-setting price in the spirit of the Calvo parameter since both the entry rate and the exit rate are constant in this model. The term  $\frac{\bar{Z}^B}{Z^A}$  can be interpreted as a markup of product B on product A.

With exogenous entry and exit rates, the model generates a constant number of products. There is no extensive margin effect in this model. Naturally, this model shows very similar dynamics against demand as New Keynesian Phillips curve by the Calvo (1983) - Yun (1996) with some differences in parameters for demand shock.

When the exit rate  $\rho$  increases, inflation is more responsive to demand shocks. This is because a chance to set a price increases when the turnover of product cycle increases. This model has another parameter related to the frictional goods market,  $b$ , which represents the bargaining power of firm A. When  $b$  decreases, the inflation rate becomes less sensitive to demand shocks since firm B can take a larger share of the surplus and is likely to keep the price of input good A unchanged against a demand shock.

In the model with endogenous entry with a fixed price, i.e.,  $g = 1$ , we have the following linearized price equation.

$$\begin{aligned} \pi_t = & \beta \mathbb{E}_t \pi_{t+1} + \beta(1-b) \frac{\bar{q}}{1 - \beta(1 - \rho - \bar{q})} \frac{\rho [1 - \beta(1 - \rho)]}{1 - \rho} \hat{\theta}_t \\ & + b \frac{\rho [1 - \beta(1 - \rho)]}{1 - \rho} \frac{\bar{Z}^B}{Z^A} \hat{Z}_t^B. \end{aligned} \quad (16)$$

We can observe an explicit effect of goods market frictions through the market tightness  $\hat{\theta}_t$ . This generates a direct link between product cycles and prices. When the demand for goods changes, the entry rate by firm B changes. Therefore, the number of products in the market also changes. More importantly, the number of new matches would adjust and it implies that the fraction of products changing prices would adjust accordingly. These behaviors are summarized in the market tightness. One way to interpret our results is that the model endogenizes the Calvo parameter through a search and matching product market. In this sense, we argue that there is an extensive margin effect

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is one.

associated with a price change. In contrast to Bilbiie, Ghironi, and Melitz (2007) where the number of products in their Phillips curve is originally included in a price index, i.e., variety effect, and an aggregate output, our model includes the market tightness from a friction in a goods market. The market tightness is positively related to a price and increases a price volatility.<sup>18</sup> Regarding effect of parameters on price dynamics, an exit rate  $\rho$ , a matching probability  $\bar{q}$ , and the bargaining power of firm  $A$   $b$  decide a response of an inflation rate to the market tightness in (16).

In details, the goods market friction captured by  $\hat{\theta}_t$  accelerates/decelerates price dynamics as shown in the following equation.

$$\hat{\theta}_t = \beta(1 - \rho - \frac{b}{\alpha}\bar{q})\mathbb{E}_t\hat{\theta}_{t+1} + (1 - b)\frac{1 - \beta(1 - \rho)}{\alpha}\frac{\bar{Z}^B}{\bar{Z}^B - Z^A}\mathbb{E}_t\hat{Z}_{t+1}^B. \quad (17)$$

The market tightness  $\hat{\theta}_t$  depends on the demand shock. Goods market frictions allow the market tightness to adjust, which further changes the price dynamics. Here the two equations above can describe price dynamics in this model. When the exit rate  $\rho$  increases, the market tightness is relatively more sensitive to a demand rather than the future market tightness due to a quicker product cycle. The bargaining power of firm  $A$   $b$ , a matching elasticity  $\alpha$ , and a matching probability  $\bar{q}$  also decide dynamics of the market tightness.

For a model with price discounting with  $g < 1$ , we have the following linearized price equation.

$$\begin{aligned} \pi_t = & \beta\mathbb{E}_t\pi_{t+1} + \frac{1 - g}{(1 - \rho)g}(\hat{m}_t - \hat{n}_t) - \beta(1 - g)\mathbb{E}_t(\hat{m}_{t+1} - \hat{n}_{t+1}) \\ & + (1 - b)\beta\bar{q}\bar{S}\frac{1 - \beta(1 - \rho)g}{Z^A}\frac{1 - (1 - \rho)g}{(1 - \rho)}\left[\hat{\theta}_t + \frac{(1 - g)}{g}\mathbb{E}_t\sum_{j=0}^{\infty}\beta^j(1 - \rho)^j\hat{\theta}_{t+j}\right] \\ & + b\frac{1 - (1 - \rho)g}{g}\frac{1 - \beta(1 - \rho)g}{1 - \rho}\frac{\bar{Z}^B}{Z^A}\hat{Z}_t^B, \end{aligned}$$

where  $M_t = q_{t-1}u_{t-1}$  denotes the number of new products and so the term  $\hat{m}_t - \hat{n}_t$  expresses a share of new products in total products, i.e., an entry rate. The effect of

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<sup>18</sup>In the general equilibrium model that we develop in Section 7, we incorporate the variety effect through a price aggregator. The variety effect works to decrease inflation rate though the market tightness works to increase an inflation rate. Thus, these two effect are sharply different in general.

a product entry is explicitly included in a model. The inflation rate can increase when the entry rate increases since the effect of price discounting for existing goods in the aggregate price relatively decreases to the that of a new price. Moreover, a price response to demand increases when  $g$  is less than one since firms have incentive to set a higher first price due to price decline after entry as observed in a model before log-linearization. This effect directly increases a volatility of a first price. This mechanism increases a ratio of a standard deviation of the first price to that of an average price. Also, the future market tightness works to amplify price dynamics and to increases persistence of the inflation rate.

## 5 Analysis for Nikkei Data

Now, we show a performance of our model using product data in Japan.<sup>19</sup> We calibrate a model with exogenous entry, a model with endogenous entry and  $g = 1$ , and a model with a price discounting with  $g = 0.944$  by quarterly base as shown in Table 3. The discount rate is 0.99 as in conventional models and the exit rate is  $\rho = 0.11$  from data. We set  $\alpha = 0.122$  from Basic Survey on Commercial and Manufacturing Structure and Activities in Japan. This number is given by a ratio of the number of producers to retailers.<sup>20</sup> We set the entry cost  $\kappa$  and  $\chi$  to achieve a steady state value of  $\theta_t$  as 15.9 that is based on Basic Survey on Commercial and Manufacturing Structure and Activities and a steady state value of  $N_t$  as 0.66 in which we assume two standard deviation above an average value as the maximum number of match of  $N_t$  and then calculate this utilization

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<sup>19</sup>When we apply our model for prices that consumers face, we assume that prices for consumers change parallel to prices for firms. For example, a firm  $B$  sets a regular price included in  $Z_t^B$  for consumers by putting a constant markup on a price of  $Z^A$ . When a firm  $B$  negotiates a price of  $Z^A$  with a firm  $A$ , a price for consumers is exogenously given since a firm  $B$  considers a possibility for sales ex ante.

<sup>20</sup>This is calculated as the number of producers for foods and drinks over the number of retailers for foods, drinks, and tobaccos plus the number of producers for foods and drinks.

rate.<sup>2122</sup> In a price discounting model,  $g$  is 0.944 to make a difference between a new price and an average price in data. We give a positive demand shock with a persistence of 0.9 estimated by data and match its standard deviation between a model with price discounting and data. The bargaining power of firms is  $b = 0.5$  so that sellers and buyers hold an equal bargaining power.

Table 4 shows simulation results. The fourth column shows the case of a model with price discounting with  $g = 0.944$ . We also show the case of  $g = 1$  as a model with endogenous entry in the third column and the model with exogenous entry in a second column.

The model with exogenous entry does not include a variable entry rate nor goods number so that we cannot observe relationships of an entry rate and the number of goods with other variables. The ratio of a new price average to an average price is always 1 because there is no mechanism for new prices to deviate from existing prices. For the ratio of the standard deviation of a new price to the standard deviation of an average price, the model with exogenous entry explains 60 percent of data. This model is like New Keynesian Phillips curve with the Calvo's price adjustment and therefore well shows a positive correlation between demand and price as 0.75 in our model and 0.87 in data.

The model with endogenous entry replicates pro-cyclicality between entry and demand. This model, however, cannot generate pro-cyclicality between entry and price since variations of new prices are not sufficient to change average price as explained below. In the model, the correlation between entry and demand (price) is 0.22 (−0.06) compared to 0.15 (0.14) in data. For the number of goods, the correlation between the number of goods and demand (price) is 0.62 (0.64) compared to 0.82 (0.74) in data.

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<sup>21</sup>To calculate  $\bar{\theta}$ , we first calculate  $\bar{v}$  from a following equation.

$$\frac{\bar{v} + (1 - \rho)\bar{N}}{\text{the maximum number of } u_t} = \frac{\text{the number of retailers}}{\text{the number of producers}}, \quad (18)$$

where the maximum number of  $u_t$  is one. Then, we can calculate  $\bar{u}$  and  $\bar{\theta}$ .

<sup>22</sup>In this parameter set, we calculate  $\frac{Z^B}{X} = 1.71$ . This is consistent with the number of  $\frac{Z^B}{X} = 1.62$  from From Basic Survey on Commercial and Manufacturing Structure and Activities.

When there is a positive demand shock, it raises the benefits for firm B to enter the market. As more firm B enter the market, the total number of matches increases. Moreover, in each new match, the positive demand shock raises total trading surplus which leads to a higher new price. As a result, an average price of good  $A$  increases. It follows that the correlation between prices and demand is also positive 0.75 in the model and 0.87 in data. The model is able to capture the key aspects of product cycle and prices. For the ratio of standard deviation of a new price to standard deviation of an average price, the model with endogenous entry gives 1.42 compared with 2.34 in data. Thus, the model with endogenous entry explains 61 percent of data and the variation in new prices is not large enough. Owing to goods market frictions, the standard deviation of price increases 31 percent in the model with endogenous entry from the model with exogenous entry.

In the model with price discounting, we can observe pro-cyclicality between entry and demand/price. The correlation between the entry rate and demand (price) is 0.22 (0.06) compared to 0.15 (0.14) in data. The correlation between the number of goods and demand (price) is 0.62 (0.67) compared to 0.82 (0.74) in data. The correlation between demand and prices is 0.84, which is also very close to 0.87 from the data. The advantage of this model is that there is a mechanism for new prices to deviate from existing prices. It implies that the average new price can deviate from average price, with the ratio of a new price average to an average price 1.4 compared to 1.38 from data. For the ratio of standard deviation of a new price to standard deviation of an average price, the model gives 1.81 compared to 2.34 in data. Thus, the model with price discounting explains 77 percent of data. Goods market frictions and price discounting raises the standard deviation of price by 45 percent compared to the model with exogenous entry. It shows that the model with price discounting perform the best among three models it replicates all features of data.

## 6 Quantitative General Equilibrium Model

The simple model is a partial equilibrium model where firm  $A$ 's cost of production  $X_t$ , quantity traded in each match  $Z^A$  and firm  $B$ 's benefit from trading  $Z_t^B$  are all exogenous. In this section, we extend the simple model of product cycle to a general equilibrium model by endogenizing  $X_t$ ,  $Z^A$  and  $Z_t^B$ . In addition to firm  $A$  (intermediate goods producers) and firm  $B$  (final goods producers), we introduce a representative household and a central bank into the model.<sup>23</sup>

The representative household has the standard love-of-variety preference and purchases a variety of goods from final goods producers. As usual, the household optimally makes intertemporal decisions on the demand for the aggregate consumption basket, the amount of asset holdings and labor supply. We assume that each final goods producer carries a distinct variety. Final goods producers set the price of each variety following the standard monopolistic competition structure. To acquire goods to sell to households, final goods producers (firm  $B$  in the simple model) search for intermediate goods producers (firm  $A$  in the simple model) in the frictional goods market. The structure of the frictional product market remains the same as in the simple model, where the first price is set through bargaining. To close the model, we assume that the central bank sets the return to assets, i.e., the interest rate, following a Taylor type rule.

We again consider several versions of the model depending on our assumptions about entry and how subsequent prices evolve after the first price is set in a match. The first version of the general equilibrium model assumes that entry into the product market by final goods producers is exogenous and subsequent prices do not change after the first price. In the second version, we endogenize entry decisions by final goods producers and can examine how endogenous product cycle affects prices and inflation. In the third version, we maintain the endogenous product cycle and assume price discounting after first prices, where the price discounting factor is exogenous. Lastly, the fourth version modifies the third version by allowing prices to decline in response to changes in the

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<sup>23</sup>See Appendix E for details of a general equilibrium model.

number of goods. In particular, for a demand shock, the number of goods increases so prices of existing goods can decrease due to depreciation of quality/preference. This can capture a fashion effect.

As for our calibration strategy, we borrow parameters from the simple model that can correctly replicate Nikkei Data at product level as in Table 3. In addition to micro level parameters, we set macro level parameters as shown in Table 5. We use conventional values for Japanese economy following Sugo and Ueda (2008). We set an inverse of elasticity of intertemporal substitution as  $\sigma = 1.249$ , an inverse of elasticity of labor as  $\phi = 2.149$ , and a goods substitution as  $\varepsilon = 6$ . For the monetary policy rule, we assume the Taylor rule as shown in Sugo and Ueda (2008). We give demand shocks with a standard deviation of a demand gap as  $\sigma_C = 0.048$  and its persistence as  $\rho_C = 0.9$  in the IS equation.<sup>24</sup> Regarding price changes after entry, we assume a negative shock of  $g_{t-1,t}$  to replicate a price decline from  $t - 1$  to  $t$  for existing goods and calibrate the standard deviation of shock  $\sigma_g = 0.076$  and its AR(1) persistence  $\rho_g = 0.8$ .<sup>25</sup> In this case, an average price level decreases 38 percent as observed in micro product level data for one standard deviation shock. At the same time, as an alternative case, we assume that a price change after entry depends on the number of goods as  $g_{t-1,t} = -N_t$ .

The following analytical result in this general equilibrium model can be useful to understand simulation outcomes. In particular, the Phillips curve with endogenous product cycles is given by

$$\pi_t^H = \beta \mathbb{E}_t \pi_{t+1}^H + \kappa_\theta \hat{\theta}_t + \kappa_C \hat{C}_t - \kappa_N \hat{N}_t + \kappa_{g1} \hat{g}_{t-1,t} - \kappa_{g2} \mathbb{E}_t \hat{g}_{t,t+1}, \quad (19)$$

where  $\pi_t^H$  is an inflation rate in a general equilibrium model and  $\kappa_\theta$ ,  $\kappa_C$ ,  $\kappa_N$ ,  $\kappa_{g1}$ , and  $\kappa_{g2}$  are parameters. For reasonable parameters including those in Table 5, these parameters are positive.

This Phillips curve is similar to one derived from the simple model with product

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<sup>24</sup>The demand gap is calculated as a deviation of a demand level from its trend in our micro data. Here, we calculate a trend by using HP filter with a smoothing parameter as 1600.

<sup>25</sup>This declining speed is set to match a price decline in Figure 2.

cycles. The price depends on the market tightness and the demand for goods.  $\hat{g}_{t-1,t}$  and  $\hat{g}_{t,t+1}$  are price discounting factors from  $t - 1$  to  $t$  and from  $t$  to  $t + 1$ , respectively. One difference from the simple model is that variety of goods negatively contributes to prices as in Bilbiie, Ghironi, and Melitz (2007). This is a variety effect of goods from the price aggregator with a love-of-variety preference. Moreover, a fundamental difference from the simple model regardless of similarity in the price equation is that this Phillips curve comes with other endogenous variables such as consumption, market tightness, the number of goods, and the policy rate. Therefore, the general equilibrium model includes feedback effects among these variables, which are absent in the simple model.

Table 6 shows simulation results. The presence of goods market frictions raises the standard deviation of inflation to the same demand variation as shown in the third row. Feedback effects among variables amplify the role of goods market frictions. From the model with exogenous entry to the model with endogenous entry, the standard deviation increases by 20 percent. Moreover, the standard deviation of inflation increases by 72 percent in the model with price discounting from the model with exogenous entry. The model with price discounting by the number of goods also sufficiently increases the standard deviation of inflation by 49 percent. These results imply that goods market frictions generates an essential mechanism to explain price dynamics in Japan.

Even when we exclude the variety effect in the model with endogenous entry, the standard deviation of inflation slightly changes to 0.23 from 0.24 in the original model with endogenous entry. For the Japanese economy, the variety effect on inflation is very marginal.<sup>26</sup>

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<sup>26</sup>Unlike Bilbiie, Ghironi, and Melitz (2007), we do not assume that an entry cost consumes resource. This point can be important for our result.



## 7 Extension

### 7.1 Analysis for Klenow and Kryvtsov (2008)

Several previous papers focus on details of price changes using micro scanner data including observations for product entry and exit. For example, Klenow and Kryvtsov (2008) use U.S. data of the Bureau of Labor Statistics for the Consumer Price Index and give several facts for a product cycle and price. They also show whether conventional price models can explain such facts or not. We show performance of our model to these facts. We use a model with endogenous entry and  $g = 1$ .

#### 7.1.1 Product Cycle with Items Substitutions

In calibration, we pick up an exit rate from Klenow and Kryvtsov (2008) and also use conventional values for parameters as in Table 7. We set a discount rate as 0.997 as in conventional models for monthly base. For an exit rate, we set  $1/9.3$  from price duration of adjacent prices in Klenow and Kryvtsov (2008).<sup>27</sup> Adjacent price captures a price change for consecutive monthly regular prices between substitutions. Here, substitutions also include stockout and out-of-season items in addition to product creation and destruction. This is interpreted as a product cycle in a shop level. For  $\alpha$  and  $b$ , we arbitrary set these numbers to match a model to data. In particular, we need a high value for a matching elasticity for a goods demand as  $\alpha = 0.9$ . Thus, we assume that the number of match is more elastic to goods demand than goods supply. Regarding  $b$ , we assume the same bargaining power for demand and supply sides as  $b = 0.5$ . For scale parameters such as  $\chi$ ,  $k$ ,  $\bar{Z}^B$ ,  $\bar{X}$ , and  $Z^A$ , we arbitrary set these numbers. These parameters do not change simulation outcomes. We give a positive demand shock with a monthly persistence of 0.55 and set a size of shock to match a standard deviation of an inflation rate as 0.0036 as given by Klenow and Kryvtsov (2008).<sup>28</sup>

Table 8 shows simulation results as Case 1 and statistics from Klenow and Kryvtsov

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<sup>27</sup>We pick up  $1/9.3$  (median) from adjacent prices in Table II of Klenow and Kryvtsov (2008).

<sup>28</sup>Note that we have similar outcomes even when we change a persistence of shock.

(2008). Our model well replicates aspects of product cycle and price change. In particular, a correlation between an inflation rate and a fraction of a price change is 0.33 in the model and 0.25 in data.<sup>29</sup> Regarding the standard deviation of a fraction of a price change, the model shows 0.031 compared to 0.032 in data. Note that all price fluctuations are given by a first price and a model ignores other price changes. Thus, we need to carefully interpret outcomes. Shapiro and Wilcox (1996) show that product substitutions can explain about half of the inflation rate. When we follow their outcomes, a model explains about 50 percent of a variation of a fraction of a price change. This outcome is consistent with Bils and Klenow (2004). They reveal that price changes by substitutions sufficiently change overall frequency of price changes.

### 7.1.2 Product Cycle with Regular Price Change

We use a different exit rate as  $1/7.2$  in Case 2.<sup>30</sup> This is a number for a median of frequency of a regular price change excluding price change by sales. In this case, we assume that a firm interprets an existing good in a shop as a new goods for a new customer with a price negotiation when firm changes a price. This is a simple way to capture price changes after a first price.

In simulation, we use parameters as shown in Table 7. A third column of Case 2 in Table 8 shows outcomes. Our model well replicates aspects of product cycle and price change observed in data. A correlation between an inflation rate and a fraction of a price change is 0.38 in the model and 0.25 in data. For the standard deviation of a fraction of a price change, the model performs very well as 0.029 compared to 0.032 in data.

### 7.1.3 Product Cycle with Forced Item Substitutions

We assume a smaller exit rate as 0.03. Klenow and Kryvtsov (2008) imply that the monthly rate of forced item substitutions is about 3 percent in their sample. In this

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<sup>29</sup>In our model, a fraction of a price change is given by a ratio of the number of new goods to the number of all goods since firms set a price only when they match in a market with a new good.

<sup>30</sup>We pick up  $1/7.2$  (median) from regular prices in Table II of Klenow and Kryvtsov (2008).

case, price changes by goods substitutions through stockouts and seasonal changes are excluded. This is a product cycle in a production level with goods creation and destruction.

In a simulation, we use parameters in Table 7. A fourth column of Case 3 in Table 8 shows outcomes. Our model can not explain a positive correlation between an inflation rate and a fraction of a price change. A model shows  $-0.14$  to  $0.25$  in data. This implies that a product cycle in store level rather than product level holds more important role for price dynamics. For a standard deviation of a fraction of a price change, a model still perform well and shows  $0.031$  to  $0.032$  in data.

## 8 Concluding Remark

We build a new price model with a frictional product market. Product cycles naturally emerge by explicitly modeling product entry and exit. Endogenous product cycles are accompanied by price cycles, where first prices can be set in different manners from subsequent prices. Our model generates a New Keynesian Phillips curve as a special case and shows that product market frictions help explain price dynamics. We calibrate our model using the product level POS data in Japan. We show that our model performs well to explain observations related to product cycles and price cycles. In a general equilibrium model, we find that an endogenous product entry can amplify the standard deviation of the inflation rate by 20 percent. A price discounting after a first price further increases this number to 72 percent.

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Table 1: Statistics for Entry Rate, Exit Rate, and Prices

|                                 | Average | Standard deviation |
|---------------------------------|---------|--------------------|
| Entry rate                      | 0.12    | 0.023              |
| Exit rate                       | 0.11    | 0.012              |
| New price average/Average price | 1.38    | 2.33               |

Note: Quarter base. For a standard deviation of New price average/Average price, we shows a standard deviation of new price average over a standard deviation of average price.

Table 2: Correlations

| Frequency                     | Quarterly | Yearly |
|-------------------------------|-----------|--------|
| Corr(entry rate, price)       | 0.14      | 0.41   |
| Corr(entry rate, demand)      | 0.15      | 0.48   |
| Corr(number of goods, price)  | 0.74      | 0.79   |
| Corr(number of goods, demand) | 0.81      | 0.85   |
| Corr(price, demand)           | 0.87      | 0.8    |

Note: Corr denotes correlation between two variables.



Table 3: Calibrations for Nikkei Japanese Data

| Parameters   | Explanations                 | Values |
|--------------|------------------------------|--------|
| $\beta$      | Discount factor              | 0.99   |
| $\rho$       | Exit rate                    | 0.11   |
| $g$          | Price discounting rate       | 0.944  |
| $\rho_{Z^B}$ | Shock persistence            | 0.9    |
| $\alpha$     | Matching elasticity          | 0.122  |
| $b$          | Firm $A$ 's bargaining power | 0.5    |
| $\chi$       | Matching efficiency          | 0.141  |
| $k$          | Entry cost by firm $B$       | 0.03   |
| $\bar{Z}^B$  | Firm $B$ 's benefit          | 2.48   |
| $\bar{X}$    | Firm $A$ 's cost             | 1.45   |
| $Z^A$        | Firm $A$ 's production       | 1      |

Table 4: Simulation Statistics for Nikkei Japanese Data

|                                   | Data | Exog. entry | Endo. entry | Price disc. |
|-----------------------------------|------|-------------|-------------|-------------|
| Corr(entry rate, price)           | 0.14 | n.a         | -0.06       | 0.06        |
| Corr(entry rate, demand)          | 0.15 | n.a         | 0.22        | 0.22        |
| Corr(number of goods, price)      | 0.74 | n.a         | 0.64        | 0.67        |
| Corr(number of goods, demand)     | 0.82 | n.a         | 0.62        | 0.62        |
| Corr(price, demand)               | 0.87 | 0.75        | 0.75        | 0.84        |
| Std(average price)                | 0.52 | 0.36        | 0.47        | 0.52        |
| New price average/Average price   | 1.38 | 1           | 0.97        | 1.4         |
| Std(new price)/Std(average price) | 2.34 | 1.41        | 1.42        | 1.81        |

Note: Quarterly base. Corr denotes a correlation between two variables. Std denotes a standard deviation.

Table 5: Calibrations for General Equilibrium Model

| Parameters           | Explanations                                  | Values |
|----------------------|---|--------|
| $\sigma$             | Inverse of elasticity of substitution         | 1.249  |
| $\phi$               | Inverse of elasticity of labor                | 2.149  |
| $\varepsilon$        | Goods substitution                            | 6      |
| $\delta_\pi$         | Coefficient for inflation rate                | 0.606  |
| $\delta_C$           | Coefficient for the output gap                | 0.11   |
| $\delta_{\Delta\pi}$ | Coefficient for a change of inflation rate    | 0.25   |
| $\delta_{\Delta C}$  | Coefficient for a change of the output gap    | 0.647  |
| $\delta_i$           | Coefficient for interest rate lag             | 0.842  |
| $\sigma_C$           | Standard deviation of demand shock            | 0.048  |
| $\rho_C$             | Persistence of demand shock                   | 0.9    |
| $\sigma_g$           | Standard deviation of price discounting shock | 0.076  |
| $\rho_g$             | Persistence of price discounting shock        | 0.8    |

Table 6: Simulation Statistics for General Equilibrium Model

|                      | Exog. entry | Endo. entry | Price disc. | Price disc. by $N$ |
|----------------------|-------------|-------------|-------------|--------------------|
| Std(inf)             | 0.016       | 0.018       | 0.043       | 0.021              |
| Std(demand)          | 0.078       | 0.074       | 0.12        | 0.07               |
| Std(inf)/Std(demand) | 0.2         | 0.24        | 0.35        | 0.3                |

Note: Quarterly base. Std denotes a standard deviation. Std denotes a standard deviation and inf denotes an inflation rate.

Table 7: Calibrations for Klenow and Kryvtsov (2008)

| Parameters   | Explanation                  | Values                 |
|--------------|------------------------------|------------------------|
| $\beta$      | Discount factor              | 0.997                  |
| $\rho$       | Exit rate                    | 1/9.3 or 1/7.2 or 0.03 |
| $g$          | Price discounting rate       | 1                      |
| $\alpha$     | Matching elasticity          | 0.9                    |
| $b$          | Firm $A$ 's bargaining power | 0.5                    |
| $\rho_{Z^B}$ | Shock persistence            | 0.55                   |
| $\chi$       | Matching efficiency          | 0.4                    |
| $k$          | Entry cost by firm $B$       | 1                      |
| $\bar{Z}^B$  | Firm $B$ 's benefit          | 2                      |
| $\bar{X}$    | Firm $A$ 's cost             | 1                      |
| $Z^A$        | Firm $A$ 's production       | 1                      |

Table 8: Simulations for Klenow and Kryvtsov (2008)

|                                | Klenow and Kryvtsov | Case 1 | Case 2 | Case 3 |
|--------------------------------|---------------------|--------|--------|--------|
| Corr(inf, frac)                | 0.25                | 0.33   | 0.38   | -0.14  |
| Std(freq price)                | 0.032               | 0.031  | 0.029  | 0.031  |
| Mean(freq price) (calibration) | 0.266               | 0.107  | 0.138  | 0.03   |
| Std(inf) (fitting data)        | 0.0036              | 0.0036 | 0.0036 | 0.0036 |

Note: Monthly base. Corr(inf, frac) denotes a correlation between an inflation rate and a fraction of price change. Mean(freq price) denotes mean of frequency of a price change. Std(freq price) denotes a standard deviation of frequency of a price change. Std(inf) denotes a standard deviation of an inflation rate.

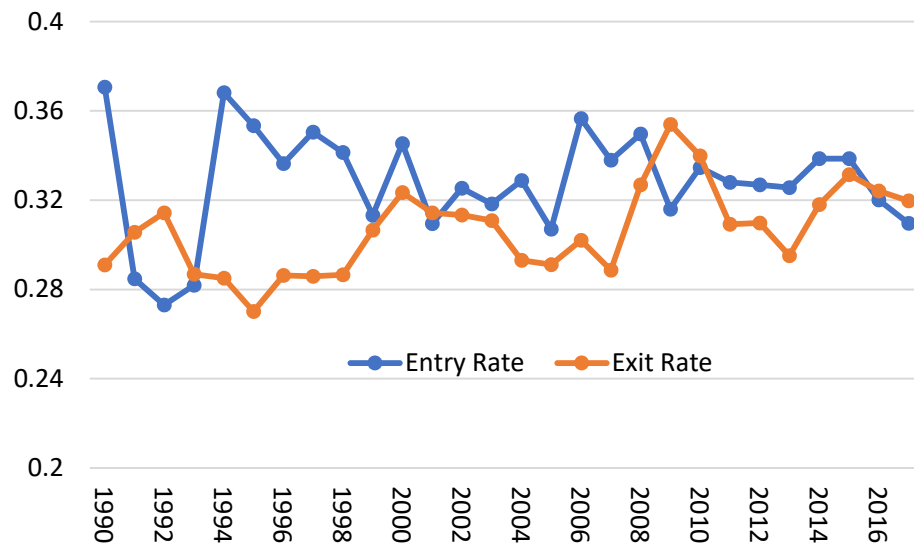


Figure 1: Entry Rate and Exit Rate

Note: Yearly base.

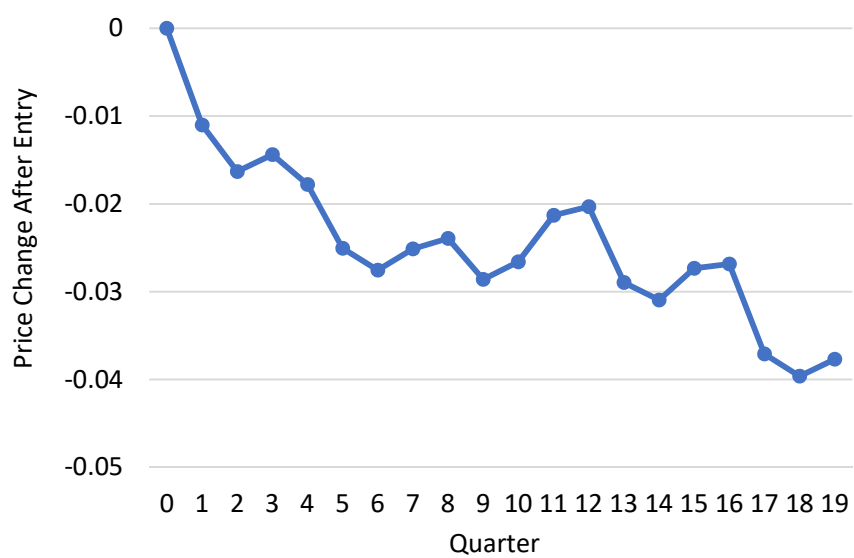


Figure 2: Price After Entry

Note: Quarterly base. Growth rate is from a first period. Note that products are restricted to those with life span of 20 quarters or more and price includes temporary sales.



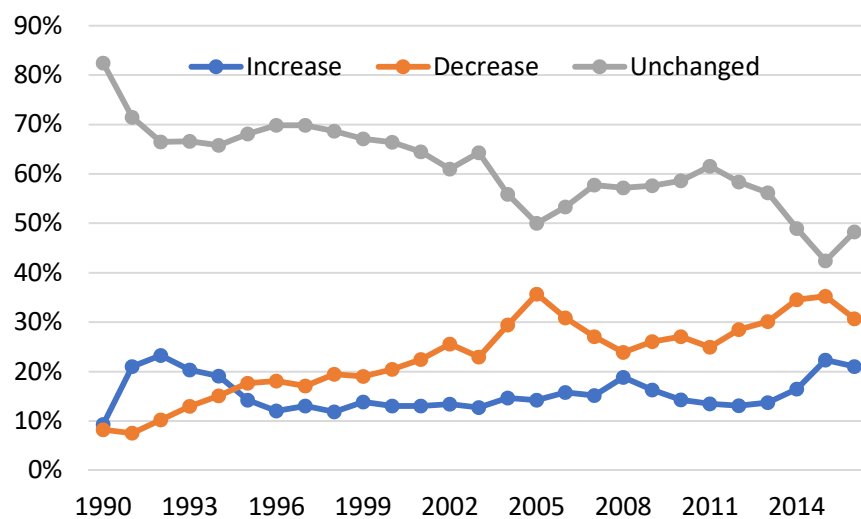


Figure 3: Ratio of No Price Change, Price Decrease, and Price Increase

Note: Yearly base. Ratio in the number of goods.

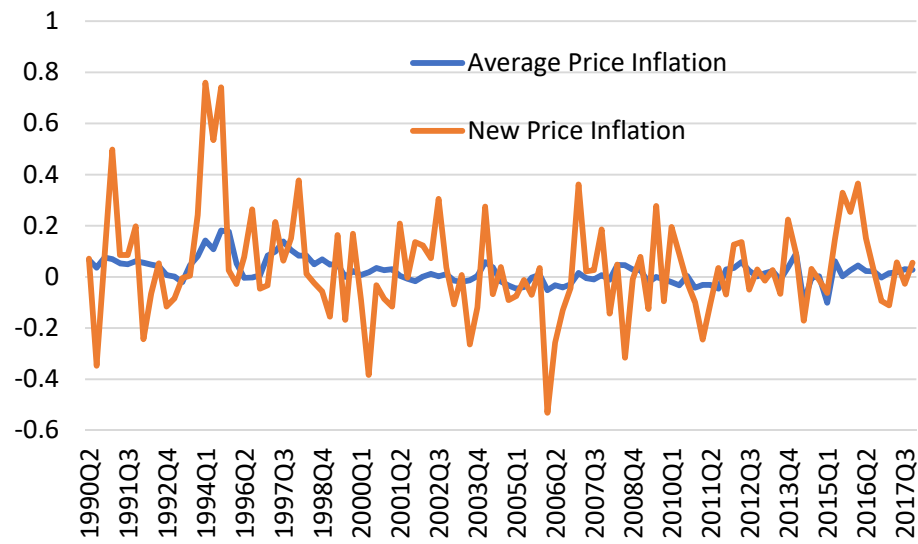


Figure 4: New Price and Average Price

Note: Quarterly base. Average price inflation denotes a year to year change of an average price. New price inflation denotes a year to year change of an average of first prices.

# Appendix

Further details of derivations of a model are in a technical Appendix.

## A Details of Data

In Nikkei data, we have the amount of sale and the quantity sold for each product in each shop on a daily basis. By dividing the amount of sale by the quantity sold, we calculate daily prices. These daily prices fluctuate due to sales promotions so that we define a modal price in a quarter or in a year as a regular price of each product in each shop. Based on regular prices, we calculate an average price, an average price for new products, and an average price for existing products.

To calculate an average price across products and shops, we use price levels. The first reason for it is that this is an average price that consumers face in shops. The second reason is that price dispersion is not large since prices in Nikkei data are those of products in supermarkets where food products and daily necessities are sold. Figure A1 shows a price distribution, where prices are defined as a yearly modal price of each product sold in each shop. The figure shows that about 70 percent of prices is between 100 yen and 999 yen. Median price and mean price are 284 yen and 622.3 yen, respectively. Minimum price and maximum price are 1 yen and 80290 yen, respectively.

## B A Model with a Price Discounting

We would like to capture the feature from the data that prices decline after the first prices. A simple way to capture price discounting is to let price decrease at a constant rate. Suppose the first price that is set in a match is  $\tilde{P}_t^A$  in period  $t$ . If the match survives in period  $t + 1$ , the price of good  $A$  becomes  $g\tilde{P}_t^A$  in this match, where  $g \in (0, 1]$ . If the same match survives in period  $t + 2$ , the price of good  $A$  declines to  $g^2\tilde{P}_t^A$  in the same match. Over time, we observe a declining price cycle.

Firm  $B$ 's free entry condition remains as (4). The value of a matched firm  $B$  is

$$\begin{aligned} V_t(P_t^A) &= Z_t^B - Z^A \tilde{P}_t^A + \beta(1-\rho) \mathbb{E}_t V_{t+1}(\tilde{P}_t^A) \\ &= Z_t^B - Z^A \tilde{P}_t^A + \beta(1-\rho) \mathbb{E}_t \left\{ Z_{t+1}^B - Z^A g \tilde{P}_t^A + \beta(1-\rho) \mathbb{E}_{t+1} V_{t+2}(\tilde{P}_t^A) \right\}. \end{aligned}$$

Notice that  $g \tilde{P}_t^A$  is the price of good  $A$  in period  $t+1$ . If we iterate  $V_t(\tilde{P}_t^A)$  forward and use  $V_{t+1}(\tilde{P}_{t+1}^A)$ , we have

$$V_t(P_t^A) = Z_t^B - \frac{Z^A}{1-\beta g(1-\rho)} \tilde{P}_t^A + \frac{\beta(1-\rho) Z^A}{1-\beta g(1-\rho)} \mathbb{E}_t \tilde{P}_{t+1}^A + \beta(1-\rho) \mathbb{E}_t V_{t+1}(\tilde{P}_{t+1}^A). \quad (20)$$

For a newly matched firm  $A$  and an unmatched firm  $A$ , the value functions are the same as (6) and (7). The benefit of having a match can be expanded as

$$\begin{aligned} & J_t^1(\tilde{P}_t^A) - J_t^0 \\ &= Z^A \tilde{P}_t^A - X_t + \beta \mathbb{E}_t \left\{ (1-\rho) \left[ J_{t+1}^1(\tilde{P}_t^A) - J_{t+1}^0 \right] - q_t \left[ J_{t+1}^1(\tilde{P}_{t+1}^A) - J_{t+1}^0 \right] \right\} \\ &= Z^A \tilde{P}_t^A - X_t - \beta \mathbb{E}_t q_t \left[ J_{t+1}^1(\tilde{P}_{t+1}^A) - J_{t+1}^0 \right] \\ &\quad + (1-\rho) \beta \mathbb{E}_t \left[ \begin{aligned} & Z^A g \tilde{P}_t^A - X_{t+1} \\ & + \beta \mathbb{E}_{t+1} \left[ (1-\rho) \left[ J_{t+2}^1(\tilde{P}_t^A) - J_{t+2}^0 \right] - q_{t+1} \left[ J_{t+2}^1(\tilde{P}_{t+2}^A) - J_{t+2}^0 \right] \right] \end{aligned} \right]. \end{aligned}$$

Similarly, we can expand  $J_{t+1}^1(\tilde{P}_{t+1}^A) - J_{t+1}^0$  and find the benefit from having a match

$$\begin{aligned} J_t^1(\tilde{P}_t^A) - J_t^0 &= \frac{1}{1-\beta g(1-\rho)} Z^A \tilde{P}_t^A - \frac{\beta(1-\rho)}{1-\beta g(1-\rho)} Z^A \mathbb{E}_t \tilde{P}_{t+1}^A - X_t \\ &\quad + \beta \mathbb{E}_t \left\{ (1-\rho - q_t) \left[ J_{t+1}^1(\tilde{P}_{t+1}^A) - J_{t+1}^0 \right] \right\}. \end{aligned} \quad (21)$$

The matching probabilities are given by (2) and (3). The evolution of the the number of total matches and the measure of unmatched firm  $A$  are given by (12) and (10). The Nash bargaining problem is set in the same way as before, except that

$$\begin{aligned} \frac{\partial V_t^A(\tilde{P}_t^A)}{\partial \tilde{P}_t^A} &= -\frac{Z^A}{1-\beta g(1-\rho)}, \\ \frac{\partial \left[ J_t^1(\tilde{P}_t^A) - J_t^0 \right]}{\partial \tilde{P}_t^A} &= \frac{Z^A}{1-\beta g(1-\rho)}. \end{aligned}$$

Then, the F.O.C yields

$$bV_t^A(\tilde{P}_t^A) = (1 - b) \left[ J_t^1(\tilde{P}_t^A) - J_t^0 \right]. \quad (22)$$

Lastly, the aggregate price index  $\tilde{P}_t^A$  is defined by

$$N_t P_t^A = (1 - \rho) N_{t-1} g P_{t-1}^A + \chi \theta_{t-1}^\alpha u_{t-1} \tilde{P}_t^A. \quad (23)$$

In the steady state, the same list of endogenous variables are solved by 9 equations, with (20), (21) and (23) replacing (5), (6), (7) and (13). The steady state for the aggregate price index  $P^A$  from (23) leads to

$$P^A = \frac{\rho}{1 - (1 - \rho)g} \tilde{P}^A,$$

where  $\tilde{P}^A$  is the steady state price of good  $A$  in a new match. Given that  $g \in (0, 1]$ , we have  $P^A < \tilde{P}^A$ .

## C A Model with Exogenous Entry

We need to change a value function for a firm B. Instead of a free entry condition, we have the value of a new match for a firm B is

$$\bar{J}_t^1(\tilde{P}_t^A) = Z_t^B - Z^A \tilde{P}_t^A + \beta E_t \left[ (1 - \rho) \bar{J}_{t+1}^1(\tilde{P}_t^A) + \rho \bar{J}_{t+1}^0 \right].$$

On the other hand, the value of a vacancy for a firm B is

$$\bar{J}_t^0 = \beta E_t \left[ \bar{s} \bar{J}_{t+1}^1(\tilde{P}_{t+1}^A) + (1 - \bar{s}) \bar{J}_{t+1}^0 \right].$$

These two equations imply that the surplus of a firm B from a new match is

$$\bar{J}_t^1(\tilde{P}_t^A) - \bar{J}_t^0 = \bar{S}_t = Z_t^B - Z^A \tilde{P}_t^A + \beta E_t \left\{ (1 - \rho) \left[ \bar{J}_{t+1}^1(\tilde{P}_t^A) - \bar{J}_{t+1}^0 \right] - \bar{s} \left[ \bar{J}_{t+1}^1(\tilde{P}_{t+1}^A) - \bar{J}_{t+1}^0 \right] \right\},$$

where we have a result of  $\bar{q} = \bar{s}$  in a steady state. In this case, goods market variables, such as  $q_t$ ,  $s_t$ ,  $N_t$ ,  $u_t$ ,  $v_t$ , and  $\theta_t$ , are constant since the number of products is constant even though other equations are same as a model in Section 3.1.

## D Relative Price Model

In a relative price model, the value function for a firm  $B$  with a contract of price  $\tilde{P}_t^A$  is

$$V_t \left( \tilde{P}_t^A \right) = Z_t^B - Z^A \frac{\tilde{P}_t^A}{P_t^A} + \beta (1 - \rho) \mathbb{E}_t V_{t+1} \left( g \tilde{P}_t^A \right).$$

The matched value functions for a firm  $A$  is given by

$$J_t^1 \left( \tilde{P}_t^A \right) = Z^A \frac{\tilde{P}_t^A}{P_t^A} - X_t + \beta \mathbb{E}_t \left[ (1 - \rho) J_{t+1}^1 \left( g \tilde{P}_t^A \right) + \rho J_{t+1}^0 \right].$$

## E General Equilibrium Model

### E.1 Household

#### E.1.1 Cost Minimization

A representative household first solves a cost minimization problem for differentiated goods.

$$\int_0^{N_t} y_t^* (i) h_t (i) di$$

subject to a consumption bundle given by

$$C_t^{\frac{\varepsilon-1}{\varepsilon}} = \left( \frac{1}{N_t} \right)^{\frac{1}{\varepsilon}} \int_0^{N_t} y_t^{*\frac{\varepsilon-1}{\varepsilon}} (i) di,$$

where  $y_t^* (i)$  and  $h_t (i)$  are individual demand and price for final good  $i$ , respectively,  $C_t$  is an aggregate demand, and  $N_t$  is the number of goods.

For the consumption aggregator, the appropriate consumption-based price index  $H_t$  is given by

$$H_t^{1-\varepsilon} = \frac{1}{N_t} \int_0^{N_t} h_t^{1-\varepsilon} (i) di.$$

Then, we have demand function for individual final goods.

$$y_t^* (i) = \left[ \frac{h_t (i)}{H_t} \right]^{-\varepsilon} \frac{C_t}{N_t}.$$

### E.1.2 Intertemporal Behavior

We consider a representative household that derives utility from consumption and disutility from labour supply. The household maximizes the following welfare function:

$$U_t = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^{t+s} [U(C_{t+s}, \nu_{t+s}) - V(L_{t+s}, \nu_{t+s})] \right\},$$

where  $\mathbb{E}_t$  is an expectation conditional on the state of nature at period  $t$ ,  $U(\cdot)$  is an increasing and concave function in the consumption index  $C_t$ ,  $V(\cdot)$  is an increasing and convex function in total labour supply  $L_t$ , and  $\nu_t$  is an exogenous disturbance of preference, where the steady state value of  $\nu_t$  is given by  $\bar{\nu} = 1$ . Note that the labor aggregator is distorted as a demand for goods,

$$L_t^{\frac{\varepsilon-1}{\varepsilon}} = \left( \frac{1}{N_t} \right)^{\frac{1}{\varepsilon}} \int_0^{N_t} l_t^{\frac{\varepsilon-1}{\varepsilon}}(h) dh,$$

where  $l_t(h)$  is labor supply to a firm  $h$ . Assuming that  $y_t^*(h) = l_t(h)$  as explained below, we have

$$L_t^{\frac{\varepsilon-1}{\varepsilon}} = \left( \frac{1}{N_t} \right)^{\frac{1}{\varepsilon}} \int_0^{N_t} l_t^{\frac{\varepsilon-1}{\varepsilon}}(h) dh = C_t^{\frac{\varepsilon-1}{\varepsilon}}.$$

$$\begin{aligned} C_t &= L_t \\ &= \left[ \left( \frac{1}{N_t} \right)^{\frac{1}{\varepsilon}} \int_0^{N_t} y_t^{*\frac{\varepsilon-1}{\varepsilon}}(h) dh \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= Y_t^*. \end{aligned}$$

where we assume no resource used for search in a goods market, just in the budget constraint as a lump sum tax. Note that aggregate output holds the same distortion as the consumption bundle eventually.

The budget constraint of the consumer is given by

$$H_t C_t + \mathbb{E}_t X_{t,t+1} B_{t+1} + D_t \leq B_t + (1 + i_{t-1}) D_{t-1} + W_t L_t + \int_0^{N_t} \Pi_t^F(f) df + T_t,$$

where  $B_t$  is a set of risky assets,  $D_t$  is the amount of bank deposits,  $i_t$  is the nominal deposit rate (policy rate) set by the central bank from  $t$  to  $t+1$ ,  $W_t$  is the nominal wage

for labor supply  $L_t$ ,  $\int_0^1 \Pi_t^F(f)df$  is the nominal dividend from owning the firm,  $T_t$  is a subsidy and  $X_{t,t+1}$  is the stochastic discount factor between  $t$  and  $t+1$ . We assume a complete financial market for risky assets. Thus, we have a unique discount factor and can characterize the relationship between the deposit rate and the stochastic discount factor as follows:

$$\frac{1}{1+i_t} = \mathbb{E}_t X_{t,t+1}.$$

Given the optimal allocation of consumption expenditure across the differentiated goods, the household must choose the total amount of consumption, the optimal amount of risky assets to hold, and an optimal amount to deposit in each period to maximize the welfare function. The necessary and sufficient conditions are given by

$$U_C(C_t, \nu_t) = \beta(1+i_t)\mathbb{E}_t \left[ U_C(C_{t+1}, \nu_{t+1}) \frac{H_t}{H_{t+1}} \right].$$

The household provides labors. We have the following relation:

$$\frac{W_t}{H_t} = \frac{V_L(L_t, \nu_t)}{U_C(C_t, \nu_t)} = \frac{L_t^\phi}{C_t^{-\sigma}}.$$

## E.2 Final Goods Producers

Final goods producers play two roles for a household and intermediate goods producers. Final goods producers sell differentiated final goods to a household in a standard monopolistic competition structure and buy input goods to produce final goods from intermediate goods producers in the frictional product market as in the simple model.

For a household, final goods producers solve

$$\begin{aligned} \max_{h_t(i)} \Pi &= \frac{h_t(i)}{H_t} y_t^*(i) - \frac{p_t(i)}{H_t} y_t(i) \\ &= \frac{h_t(i)}{H_t} y_t^*(i) - \frac{p_t(i)}{H_t} y_t^*(i), \end{aligned}$$

where we assume that final goods producers buy product from intermediate goods producers and use it to make final goods for household as  $y_t^*(i) = f^*(y_t(i)) = y_t(i)$ , where



$f^*(\cdot)$  is a production function of final goods producers and  $y_t(i)$  and  $p_t(i)$  are individual demand and price for a product from intermediate goods producer  $i$ .

F.O.C with respect to  $h_t(i)$  gives

$$h_t(i) = \frac{\varepsilon}{\varepsilon - 1} p_t(i)$$

where  $p_t(i)$  is given when deciding  $h_t(i)$ . Moreover, final goods producers first set  $p_t(i)$  with intermediate goods producers and decide the amount of input  $y_t^*(i)$  and so  $y_t(i)$  after setting a price of  $h_t(i)$ . We have

$$y_t(i) = \frac{1}{N_t} \left[ \frac{\varepsilon}{\varepsilon - 1} \right]^{-\varepsilon} \left[ \frac{p_t(i)}{H_t} \right]^{-\varepsilon} C_t.$$

Note that we have different demand function after time  $t$  when intermediate goods producers and so final goods producers do not change price, such as

$$y_{t,t+1}(i) = \frac{1}{N_t} \left[ \frac{\varepsilon}{\varepsilon - 1} \frac{g_{t,t+1} p_t(i)}{H_t} \frac{H_t}{H_{t+1}} \right]^{-\varepsilon} C_{t+1}.$$

where  $g_{t,t+j}$  is a price shock from time  $t$  to  $t+j$  to existing price, where  $\bar{g} = 1$  and  $g_{t,t} = 1$ .

For intermediate goods producers, final goods producers solve an optimization problem for frictional goods marker as in a simple model.

$$V_t(p_t(i)) = \frac{h_t(i)}{H_t} y_{t,t}^*(i) - \frac{p_t(i)}{H_t} y_{t,t}(i) + \mathbb{E}_t [\beta_{t,t+1} (1 - \rho) V_{t+1}(g_{t,t+1} p_t(i))],$$

where  $\beta_{t,t+s} = \beta^s \frac{\lambda_{t+s}}{\lambda_t}$  and  $\lambda_t = C_t^{-\sigma}$  is a marginal utility of consumption.

### E.3 Intermediate Goods Producers

There is a measure 1 of intermediate goods producers in the economy. The value function is

$$\begin{aligned} J_t^1(p_t(i)) - J_t^0 &= \frac{p_t(i)}{H_t} y_{t,t}(i) - W_t^* l_{t,t}(i) \\ &\quad + \beta_{t,t+1} \mathbb{E}_t \{ (1 - \rho) [J_{t+1}^1(g_{t,t+1} p_t(i)) - J_{t+1}^0] - q_t [J_{t+1}^1(p_{t+1}(i)) - J_{t+1}^0] \}, \end{aligned}$$

where  $y_{t,t}(i) = f(l_{t,t}(i)) = l_{t,t}(i)$ . and  $f(\cdot)$  is a production function of intermediate goods producer. We define  $W_t^* \equiv \frac{W_t}{H_t}$ . We define  $S_t \equiv J_t^1(p_t(i)) - J_t^0$  for following sections.

## E.4 Sharing Condition and Matchig for Goods

We assume the following sharing conditon to set a price as

$$(1 - b) S_t = b V_t.$$

The free entry condition for final goods producers is

$$s_t \mathbb{E}_t (\beta_{t,t+1} V_{t+1}) = k.$$

The matching function in the goods market is given by

$$m(u_t, v_t) = \chi u_t^{1-\alpha} v_t^\alpha \text{ where } \alpha \in (0, 1).$$

Here  $u_t$  represents the measure of intermediate goods producers that have not found a match with a final goods producer and  $v_t$  denotes the measure of available final goods producers that search in the goods market. Define the market tightness as  $\theta_t = v_t/u_t$ . The matching probabilities are

$$\begin{aligned} s_t &= \frac{m_t}{v_t} = \chi \theta_t^{\alpha-1}, \\ q_t &= \frac{m_t}{u_t} = \chi \theta_t^\alpha. \end{aligned}$$

The flow condition for the measure of matches is

$$N_t = (1 - \rho) N_{t-1} + q_{t-1} u_{t-1},$$

where  $u_t$  follows

$$u_t = 1 - (1 - \rho) N_t.$$

## E.5 Price Aggregation

$$\begin{aligned} H_t^{1-\varepsilon} &= \frac{1}{N_t} \int_0^{N_t} h_t^{1-\varepsilon}(i) di \\ &= \frac{q_{t-1} u_{t-1}}{N_t} h_{t,t}^{1-\varepsilon} + \left(1 - \frac{q_{t-1} u_{t-1}}{N_t}\right) g_{t-1,t}^{1-\varepsilon} H_{t-1}^{1-\varepsilon}, \end{aligned}$$

where a new price  $h_{t,t}$  is the same across price setters. We can decompose price of  $H_t$  into two parts, a new price and an old price since we assume that final goods producers set prices only when intermediate goods producers change prices. Thus, a new price is set only for a new goods in this model.

## E.6 Closed Economy

After log-linearizing a model around a constant steady state, we have a closed economy consisting of five variables and five equations.

Form a relation in frictional goods market, we have an equation regarding a market tightness.

$$\begin{aligned} \hat{\theta}_t = & \beta \left\{ 1 - \rho - \frac{b}{1-b} \frac{\bar{q}}{(1-\alpha) \left\{ \frac{b}{1-b} + \left[ \frac{\bar{W}^*}{A} \varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon} \right] \frac{\varepsilon}{\varepsilon-1} \right\}} \right\} \mathbb{E}_t \hat{\theta}_{t+1} \\ & + \frac{\sigma}{1-\alpha} \hat{C}_t \\ & + \left\{ \frac{\bar{C} \left( \frac{\varepsilon-1}{\varepsilon} - \frac{\bar{W}^*}{A} \right) - \frac{\bar{W}^* \bar{C}}{A} (\sigma + \phi) + \left[ \frac{\bar{W}^*}{A} \varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon} \right] \frac{\bar{C}}{\varepsilon-1}}{(1-\alpha) \bar{V} \left[ \frac{b}{1-b} + \left[ \frac{\bar{W}^*}{A} \varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon} \right] \frac{\varepsilon}{\varepsilon-1} \right]} - \frac{\sigma}{1-\alpha} \right\} \mathbb{E}_t \hat{C}_{t+1} \\ & - \frac{\bar{C} \left( \frac{\varepsilon-1}{\varepsilon} - \frac{\bar{W}^*}{A} \right) + \left[ \frac{\bar{W}^*}{A} \varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon} \right] \frac{\bar{C}}{\varepsilon-1}}{(1-\alpha) \bar{V} \left[ \frac{b}{1-b} + \left[ \frac{\bar{W}^*}{A} \varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon} \right] \frac{\varepsilon}{\varepsilon-1} \right]} \mathbb{E}_t \hat{N}_{t+1}. \end{aligned} \quad (24)$$

From a price setting behavior, we have a Phillips curve with a search foundation.

$$\begin{aligned} \pi_t^H = & \beta \mathbb{E}_t \pi_{t+1}^H + \frac{\rho}{1-\rho} \frac{1-\beta(1-\rho)}{\bar{C}} \frac{1}{(1-b) \left[ \frac{\bar{W}^*}{A} \varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon} \right] + \frac{\varepsilon-1}{\varepsilon} b} (1-b) \beta \bar{q} \bar{S} \hat{\theta}_t \\ & + \frac{\rho}{1-\rho} \left[ \frac{b}{\varepsilon} - (1-b) \left( \frac{\varepsilon-1}{\varepsilon} - \frac{\bar{W}^*}{A} - \frac{\bar{W}^*}{A} \sigma - \frac{\bar{W}^*}{A} \phi \right) \right] \frac{1-\beta(1-\rho)}{(1-b) \left[ \frac{\bar{W}^*}{A} \varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon} \right] + \frac{\varepsilon-1}{\varepsilon} b} \hat{C}_t \\ & - \frac{\rho}{1-\rho} \left[ \frac{b}{\varepsilon} - (1-b) \left( \frac{\varepsilon-1}{\varepsilon} - \frac{\bar{W}^*}{A} \right) \right] \frac{1-\beta(1-\rho)}{(1-b) \left[ \frac{\bar{W}^*}{A} \varepsilon - \frac{(\varepsilon-1)^2}{\varepsilon} \right] + \frac{\varepsilon-1}{\varepsilon} b} \hat{N}_t \\ & + \hat{g}_{t-1,t} - \beta \mathbb{E}_t \hat{g}_{t,t+1}. \end{aligned} \quad (25)$$

From a flow condition of products, we have

$$\hat{N}_t = (1-\rho)(1-\bar{q}) \hat{N}_{t-1} + \rho \alpha \hat{\theta}_{t-1}. \quad (26)$$

From the consumer side, we have the IS equation as

$$\hat{C}_t = \mathbb{E}_t \hat{C}_{t+1} - \sigma (\hat{i}_t - \mathbb{E}_t \pi_{t+1}^H) + Z_t, \quad (27)$$

where  $Z_t$  is a demand shock.

The Taylor rule is given by

$$\hat{i}_t = \delta_\pi \pi_{t-1}^H + \delta_C \hat{C}_{t-1} + \delta_{\Delta\pi} (\pi_t^H - \pi_{t-1}^H) + \delta_{\Delta C} (\hat{C}_t - \hat{C}_{t-1}) + \delta_i \hat{i}_{t-1} \quad (28)$$

Then, we have five endogenous variables of

$$\hat{N}_t, \hat{\theta}_t, \pi_t^H, \hat{i}_t, \hat{C}_t$$

and five equation above of (24), (25), (26), (27), and (28). We also have one exogenous variable  $Z_t$ . We also have  $\hat{g}_{t,t+1}$  for a price change after an entry.

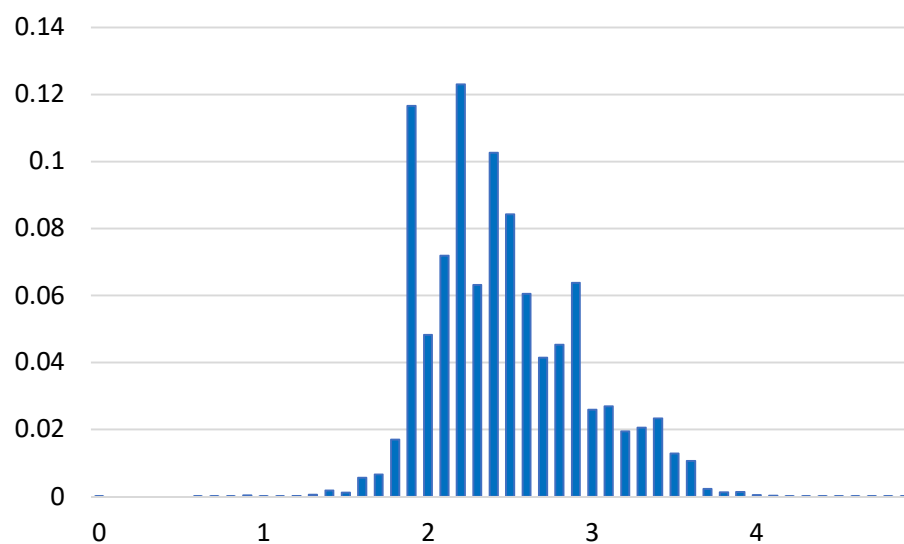


Figure A1: Price Distribution

Note: Log 10 price. Observation prices are modal prices for all products and all sample periods.